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THE

PHYSICAL REVIEW.

NOTE ON THE REVERSAL OF THE CORBINO EFFECT IN IRON.

By Alpheus W. Smith.

IN an investigation on the Hall and Corbino effects in metals and alloys Chapman[1] has made the interesting observation that the Corbino effect in iron at low magnetic fields is in the opposite direction to that at higher magnetic fields. This reversal in the direction of the Corbino effects occurs when the magnetic field which acts on the current in the plate is about 250 gausses. Now the Corbino effect is regarded as just another manifestation of the Hall effect. In the Corbino effect the electric current flowing in the circular plate is radial and the magnetic field which is perpendicular to the plane of this plate produces a current which is at right angles to the radial electric current. The electric current thus set up by the magnetic action is essentially equivalent to the transverse electric current observed in the Hall effect. If the Corbino effect reverses its direction at low magnetic fields the possibility is at once suggested that the Hall effect may also reverse its sign at these low magnetic fields. Since the direction of the Hall effect in iron has always been a serious difficulty for the theory of this effect, the possibility of the reversal of the direction of this effect at low magnetic fields is a matter of considerable importance. As all of the work on the Hall effect in iron has been done in magnetic fields which are larger than those at which this reversal of the Corbino effect was noted by Chapman, it seemed worth while to test this point by making some observations on the Hall effect in iron for smaller magnetic fields.

A large solenoid the constant of which was known was used to produce the magnetic fields. The plate was cut from a piece of pure annealed electrolytic iron. It was rectangular in form with a narrow arm projecting from the middle of each of the longer sides. To these arms were soldered the wires which were connected to the galvanometer with which the transverse electromotive force was measured. These arms were

[1] Chapman, Phil. Mag. (6), 32, p. 303, 1916.

adjusted in the manner described in an earlier paper[1] so that they were on the same equipotential line in the plate. The plate which was .0047 cm. thick was placed in the center of the solenoid so that the lines of magnetic force were perpendicular to the plane of the plate. For the measurement of the transverse electromotive forces a Leeds and Northrup high sensibility d'Arsonval galvanometer was used. This instrument gave a deflection of one mm., on a scale at a distance of 1 meter for a

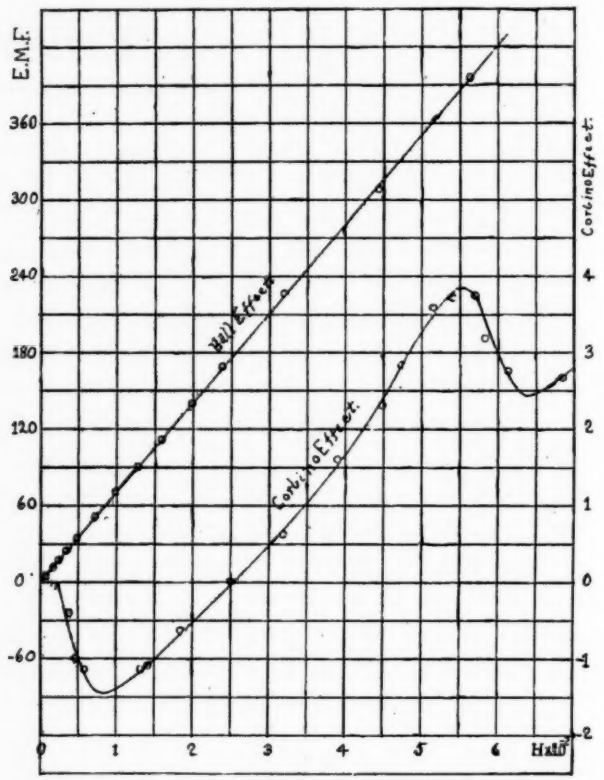

Fig. 1.

difference of potential 3.3×10^{-9} volts across its terminals. The method of taking the observations was that used by the author in his earlier work on the Hall effect. In this case the deflection of the galvanometer corresponding to a particular magnetic field was observed and the electromotive force producing this deflection was calculated from the sensibility of the galvanometer.

The results of these observations have been plotted in Fig. 1. In this figure the magnetic fields have been plotted for abscissæ and the trans-

[1] Smith, PHYS. REV., 30, p. 1, 1910.

verse electromotive forces in a plate 1 cm. thick, carrying a current of 1 electromagnetic unit have been plotted for ordinates. This electromotive force was calculated from the observed electromotive force by means of the equation $E = R(Hi/d)$, where R is the Hall constant; i the current in absolute units; d, the thickness of the plate in centimeters and E the transverse electromotive force in absolute units. In the same figure has also been plotted the curve from the paper by Chapman showing the dependence of the Corbino effect on the magnetic field. In this latter curve the ordinates are proportional to the circular current set up by the magnetic action.

This circular current is a measure of the Corbino effect and is also ordinarily a measure of the Hall effect. In this case, however, it is seen at once that the Corbino effect and the Hall effect in iron at these lower magnetic fields are not proportional to each other. The Hall effect shows no sign of reversal and is very approximately proportional to the magnetic field. On the other hand, the Corbino effect reverses its direction and does not depend in a simple way on the magnetic field. These observations shut out the possibility of the reversal of the direction of the Hall effect in iron at low magnetic fields and call attention to the marked difference in the behavior of the Hall effect and of the Corbino effect in iron.

PHYSICAL LABORATORY,
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THE RELATION BETWEEN CERTAIN GALVANOMAGNETIC PHENOMENA.

BY C. W. HEAPS.

THE various galvanomagnetic effects have for the most part been studied separately by different experimenters, or where extensive researches along these lines have been conducted it has frequently happened that conditions were such as to prevent the establishing of exact relationships among the effects studied. Nevertheless, a definite relation between some of these effects is indicated qualitatively by experiment. For example, the Hall effect is unusually large in bismuth, and correspondingly, the other related effects are found to be abnormal in this metal also. The relationship is not always simple, however. In tellurium the Hall effect is very large while the other related phenomena are not of corresponding magnitude.

The three chief galvanomagnetic phenomena are ordinarily known as the Corbino effect, the Hall effect, and the change of resistance in a magnetic field. The last two of these are comparatively well known, the first has been less extensively studied. As investigated by Corbino this phenomenon is produced when a flat circular disk in which a radial current is flowing is placed in a magnetic field. Under these conditions a circular current is set up in the plate. At first sight it appears as if this Corbino effect were nothing more than a somewhat unusual aspect of the Hall effect. In the latter phenomenon a potential difference is measured while in the Corbino effect the current resulting from the Hall potential difference is observed. In spite of this apparently simple relation experiment has up to the present time failed to give an exact quantitative relation between these two phenomena. Smith concludes[1] that "while the exact quantitative relation . . . is not obvious it is seen that the sign and order of magnitude of one effect can be predicted if the other is known." Chapman's experiments[2] seem to indicate that "there remains an outstanding difference between the two effects when measured in the same specimen." Chapman, however, did not make his experiments on the two effects in the same specimen, but in different

[1] PHYS. REV., 8, p. 402, 1916.

[2] Phil. Mag., 32, p. 303, 1916.

specimens made up from the same sample of metal, and the electrical conditions were not exactly similar in the two cases. The writer has devised a somewhat simplified method of measuring all three of the above mentioned effects in a single specimen, and the quantitative relations between them have been worked out.

Theory.

If we assume electric fields, X and Y, acting along the x and y axes, respectively, and a magnetic field, H, acting along the z axis, the equations of motion of an electron are:

$$m\frac{d^2x}{dt^2} = Xe - He\frac{dy}{dt},$$

$$m\frac{d^2y}{dt^2} = Ye + He\frac{dx}{dt}. \qquad (1)$$

Assuming Drude's simple theory we may apply these equations to the case of metallic conduction and solve them by Townsend's method.[1] They are, in fact, the same equations which Townsend solves, except for the additional term involving Y. The result obtained for the respective drift velocities, U and V, of the electrons along the x and y axes is

$$U = \frac{eT}{m(1 + \omega^2T^2)}(X - Y\omega T),$$

$$V = \frac{eT}{m(1 + \omega^2T^2)}(Y + X\omega T), \qquad (2)$$

where T is the mean free time of an electron and $\omega = H(e/m)$. These equations may be used to deduce expressions for the various galvanomagnetic effects.

Consider a conducting plate of dimensions a, b and c along the x, y and z axes, respectively. When $V = 0$ we have $Y = -X\omega T = -UH$. Let the current density along the x axis be I_x. Then[2] $I_x = \frac{2}{3}neU \cdot b \cdot c$, where n is the number of free electrons per unit volume, and

$$Y \cdot b = -\frac{3HI_x}{2nec} = \frac{RHI_x}{c}. \qquad (3)$$

The Hall coefficient, R, is thus equal to $-3/2ne$.

For the Corbino effect we consider a circular metal disk of radius r_2 in the center of which is a circular hole of radius r_1. A radial current I_r flows in the disk and a magnetic field H is normal to the plane of the disk. The potential difference, δV, between the edges of an elemental ring of

[1] Electricity in Gases, p. 100.

[2] Swann, Phil. Mag., 27, p. 441, 1914.

width δr, and distant r from the center, is given by

$$\delta V = \frac{\rho I_r \delta r}{2\pi r \cdot d}, \tag{4}$$

where d is the thickness of the disk and ρ is its specific resistance. The radial electric field is thus

$$X_r = \frac{\rho I_r}{2\pi r \cdot d}, \tag{5}$$

and since there is no applied tangential electric field equations (2) above give the tangential drift velocity of the electrons:

$$V_t = X_r \frac{e}{m} T \frac{\omega T}{1 + \omega^2 T^2}. \tag{6}$$

The tangential current in the elemental ring is

$$\delta I_t = \tfrac{2}{3} n e V_t \cdot d \cdot dr. \tag{7}$$

The radial current is given by

$$I_r = \tfrac{2}{3} n e U \cdot 2\pi r \cdot d, \tag{8}$$

from which it follows that

$$\rho = \frac{3}{2} \frac{m}{ne^2} \frac{(1 + \omega^2 T^2)}{T}. \tag{9}$$

This equation, combined with (5) and (6) above, enables us to integrate (7), giving

$$I_t = \frac{H I_r T e}{2\pi m} \log \frac{r_2}{r_1}. \tag{10}$$

This expression is the same as Corbino's when $(e/m)T$ is replaced[1] by Corbino's constant, E.

It is evident that the Hall effect and the Corbino effect yield us information about two different concepts in the theory of metallic conduction, namely the number of electrons per unit volume and the mean free time of these electrons. Hence we should not expect, in general, that there would always be correspondence between the Hall coefficient and the Corbino coefficient. To get a relation on the basis of the theory we may proceed as follows:

Consider a conducting plate of dimensions a, b and c along the x, y and z axes, respectively. Eliminating Y from equations (2) we get

$$V = \frac{X(e/m)T - U}{\omega T}. \tag{11}$$

[1] Adams finds (Phil. Mag., 27, p. 244, 1914) the constant E to be $\frac{1}{2}(e/m)T$, but his deduction assumes the average drift velocity of electrons in the direction of an electric force, X, to be $\frac{1}{2}X(e/m)T$ instead of $X(e/m)T$.

When $V = 0$ let $U = U_1$ and $X = X_1$. Then we have

$$\frac{e}{m} T = \frac{U_1}{X_1},$$

so that

$$V = \frac{1}{\omega T}\left(\frac{X U_1}{X_1} - U\right). \tag{12}$$

Since the density of the electric current is proportional to the drift velocity of the electrons we may write

$$i_y = \frac{1}{\omega T}\left(\frac{X}{X_1} i_x' - i_x\right), \tag{13}$$

where i_x and i_y are the current densities along the x and y axes, respectively, and i_x' is the value of i_x when $i_y = 0$. If two mutually perpendicular currents are sent through a metal plate by separate batteries, and if this plate is then placed in a magnetic field, i_x is equal to i_x' within the limits of ordinary observation. Equation (13) may, therefore, be put into the simplified form

$$\omega T = \frac{i_x}{i_y}\left(\frac{X - X_1}{X_1}\right). \tag{14}$$

The specific resistance of the metal plate along the x direction is given by $\rho = X_1/i_x$. It is obvious that $X - X_1$ gives the potential difference of the ordinary Hall effect, though usually this latter is measured when $X_1 = 0$. The Hall coefficient, R, is given by the relation $X - X_1 = RHi_y$, hence equation (14) becomes

$$E = \frac{e}{m} T = \frac{R}{\rho}. \tag{15}$$

Both Smith and Chapman have compared values of $E\rho$ with R for different metals and different magnetic fields. Their conclusions are as stated above. It appears, however, that there may be good reason for this failure of Smith's and Chapman's results to fit equation (15) exactly. In the first place they assume the value of ρ to be independent of H, and while this assumption may be legitimate in the case of many metals, it introduces an error where such metals as bismuth are being studied. Furthermore, by taking values of ρ from tables instead of determining them experimentally another error is likely to be introduced. It is not surprising, therefore, that they should get somewhat inexact agreement between $E\rho$ and R.

EXPERIMENT.

If a specimen is arranged in the proper way and all the quantities on the right-hand side of equation (14) are obtained experimentally then sufficient data is at hand for determining all three of the galvanomagnetic effects. Experiments were made on bismuth, copper, zinc, and natural graphite crystals. In all cases the specimen under examination was cut to the form of a fairly thin and almost square plate. This plate was then sawed into the form of a symmetrical, flat cross and each arm of the cross was divided by a number of fine saw-cuts, each cut being parallel to the cross arm in which it was made. The result was a square or rectangular plate from each edge of which extended a number of parallel arms of the same material as the plate. To the extremities of these arms were then soldered wires, the ends of which were joined together and connected in an electric circuit as shown in Fig. 1. In this diagram A and A' indicate ammeters and G is a galvanometer. In adjusting the apparatus it was found necessary to make all the wires from the ends of the arms of about the same resistance and to secure a fairly symmetrical specimen, otherwise the lines of current flow in the plate were not parallel. In order to prevent an appreciable current from flowing out of one arm through the leading-in wires and back to the plate by another arm these leading-in wires were made to have a resistance large compared with that of the plate itself.

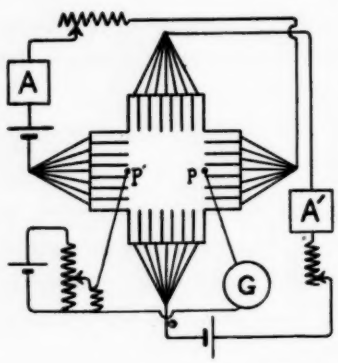

Fig. 1.

To the points p and p' other wires were soldered and led through a potentiometer circuit to the galvanometer G. The points p and p' were located so as to be on an equipotential line when the current flowed through the circuit containing the ammeter A', and the circuit containing A was open. When connections were properly made the plate was placed between the poles of an electromagnet so that the magnetic lines of force were as nearly as possible perpendicular to the face of the plate. The pole-pieces of the magnet had rectangular faces of dimensions 2.4×1.5 cm., and were in most experiments from 0.5 to 0.8 cm. apart. The strength of the magnetic field was determined by the use of a bismuth spiral.

The method of taking observations was as follows: With the electromagnet excited the ammeters A and A' were read, thus allowing the respective current densities, i_x and i_y to be calculated. The current

through the electromagnet was then broken and i_x also made zero. Under these conditions it sometimes happened that the galvanometer G showed a small deflection because of thermal electromotive forces at the junctions p and p'. By adjusting the potentiometer arrangement in series with the galvanometer this thermal E.M.F. was counterbalanced and the reading of the galvanometer again made zero. In most cases, however, there was very little effect of this kind, so that except for an initial setting to make a small compensation for inaccurate placing of the terminals, p and p', the potentiometer seldom required adjustment. The magnetizing current was next thrown on and the reading of G observed. The current i_y was next broken, the current i_x made, and the reading of G again noted. Since the galvanometer deflections are proportional to the potential drop in the plate these deflections enable the ratio $(X - X_1)/X_1$ of equation (14) to be calculated. Several sets of observations were taken by allowing i_x to flow constantly, observing the galvanometer, and noting the change in the galvanometer deflection when i_y was made or broken. These observations agreed with those obtained in the previously described manner.

In all cases the final data recorded is a mean of results obtained with i_x and i_y flowing first in one direction then in the other, though it was found that the directions of the currents had little effect on the magnitude of the results. In some cases, however, a magnetic field in one direction gave a much larger effect than when in the opposite direction. This difference was probably due to the lines of equipotential in the plate not being parallel, for it was observed that this unsymmetrical effect of the magnetic field could be largely obviated by making all connections at the ends of the arms of about equal resistance, and by making the plate itself of uniform thickness and structure.

Measurements made as described enable the quantity $(e/m)T$ of equation (14) to be calculated. To get values of ρ or R of equation (15) it was necessary to make a further observation. By means of the potentiometer it was determined what E.M.F. in the galvanometer circuit produced unit deflection of the galvanometer. Knowing this constant and the distance between p and p' the value of $X - X_1$ and of X_1 could be determined from the corresponding galvanometer deflections. Hence R and ρ could be easily calculated.

In making measurements on the Hall effect it is ordinarily considered desirable to connect the Hall electrodes to the plate with bars of the same material as the plate itself in order to minimize the effects of thermal E.M.F. For the sake of comparison the Hall effect was determined in a specimen of bismuth with the terminals of the galvanometer connected

to the respective ends of a pair of arms instead of at the points p and p'. In this case the values obtained for R were slightly larger than those previously secured. One would expect a result of this kind if some of the primary current flowing through A' passed outside the points p and p' instead of between them. However, errors arising from causes of this kind and from thermal effects were considered as small enough to be neglected in this experiment.

One other source of error must be considered in measuring the Hall effect. Properly speaking no current should be allowed to flow from the Hall terminals as a result of the potential difference set up by the magnetic field. When a current is allowed to flow through a galvanometer the assumption made in deducing equation (3), that $V = 0$, is no longer absolutely correct. Solving the second of equations (2) for Y gives

$$Y = \frac{mV}{eT}(1 + \omega^2 T^2) - X\omega T. \tag{16}$$

Let the current through the galvanometer be I_g. Then

$$I_g = \tfrac{2}{3} neV \cdot a \cdot c.$$

Hence

$$Y = \frac{3mI_g(1 + \omega^2 T^2)}{2a \cdot c \cdot ne^2 T} - X\omega T. \tag{17}$$

Applying equation (9) to this expression we get

$$Y = \frac{\rho}{a \cdot c} I_g - X\omega T. \tag{18}$$

In the experiment as performed a sensitive galvanometer of comparatively high resistance was used so that I_g was very small—in all cases the value of $\rho I_g/ac$ was small compared with the potential difference observed. Hence this term may be neglected and we get approximately the same results for Y as would have been obtained by a balancing potentiometer method. Experiment confirmed this conclusion.

Experimental Results.

The results obtained are tabulated below. The bismuth specimen was cast from the metallic crystals as supplied by Merck—about 98 per cent. pure—and the central square plate of the specimen had dimensions $1.05 \times 1.05 \times 0.20$ cm. The zinc and copper specimens were cut from sheets of the commercial metal and were of dimensions $1.6 \times 0.8 \times 0.046$ cm. and $1.35 \times 1.35 \times 0.059$ cm. respectively. The natural graphite was obtained from Orange County, N. Y., through the kindness of Mr.

A. C. Hawkins, formerly of Brown. This graphite consisted of numerous thin plates superposed upon each other, and was very perfect and free from impurities. A specimen was cemented to a glass plate with sealing wax, cut to the desired form and copper-plated at all points where wires were to be affixed by soldering. After one or two trials no difficulty was experienced in making contacts by this method, though when pressure contacts were used satisfactory results could not be obtained. The specimen used had dimensions 1.26 × 1.20 × 0.04 cm.

Copper. $\rho = 1.56 \times 10^3$ (from Baedeker's table).

H, Gausses.	E.	$R = E\rho$.	T, Seconds.	n.
5000	-2.81×10^{-7}	-4.38×10^{-4}		
9100	-2.68 "	-4.18 "		
13100	-2.65 "	-4.13 "		
14700	-2.77 "	-4.32 "		
Mean.....	-2.73×10^{-7}	4.25×10^{-4}	1.54×10^{-14}	2.15×10^{23}

Zinc. $\rho = 5.71 \times 10^3$ (from Baedeker's table).

10300	$+12.8 \times 10^{-8}$	$+7.30 \times 10^{-4}$		
13400	$+12.4$ "	$+7.08$ "		
17200	$+9.8$ "	$+5.60$ "		
Mean.....	$+11.7 \times 10^{-8}$	$+6.66 \times 10^{-4}$		

Bismuth.

H, Gausses.	E.	R.	$\rho = R/E$, C. G. S. E. M. U.	T, Seconds.	n.
800	-10.9×10^{-5}	-15.4	14.1×10^4	6.16×10^{-12}	6.09×10^{18}
2450	-6.5 "	-10.0	15.4 "	3.67 "	9.38 "
6150	-4.42 "	-7.85	17.7 "	2.50 "	11.9 "
9750	-3.12 "	-6.40	20.5 "	1.76 "	14.6 "
11100	-2.92 "	-6.15	21.1 "	1.65 "	15.2 "
12850	-2.63 "	-5.92	22.5 "	1.49 "	15.8 "

Graphite.

2600	-6.15×10^{-6}	-0.612	9.95×10^4	3.47×10^{-13}	1.53×10^{20}
4200	-6.05 "	-0.662	10.95 "	3.42 "	1.42 "
5500	-5.52 "	-0.690	12.50 "	3.12 "	1.36 "
6550	-5.52 "	-0.760	13.8 "	3.12 "	1.23 "
7600	-5.07 "	-0.780	15.4 "	2.86 "	1.20 "
8400	-4.90 "	-0.810	16.5 "	2.77 "	1.15 "
9100	-4.85 "	-0.858	17.7 "	2.74 "	1.09 "
9900	-4.60 "	-0.890	19.4 "	2.60 "	1.05 "
10550	-4.43 "	-0.910	20.5 "	2.50 "	1.03 "
11300	-4.32 "	-0.946	21.8 "	2.44 "	0.99 "

The effect to be measured was so small in zinc and copper that accurate measurements could not be made. The arrangement of apparatus is not the best for determining small changes of resistance such as occur when copper or zinc are placed in a magnetic field. Both the Hall effect and the resistance effect have been carefully measured for these metals, however, and it has been found that the Hall coefficient is independent of the magnetic field. Since the effect of H on ρ is very small in these metals we should expect the Corbino constant to be practically independent of H—a conclusion which the above data appear to corroborate in the case of copper. The variations of E in zinc are no larger than the experimental errors. It should be noted that both the Corbino effect and the Hall effect in zinc have a sign opposite to that predicted by the theory. In the other metals studied the sign is such as to make ωT in equation (14) have a positive value.

The data in the case of bismuth show that both the Hall coefficient and the Corbino constant diminish in magnitude as the strength of the magnetic field is increased. The value of E diminishes more rapidly than R—a result to be expected when the resistance of a metal is increased by a magnetic field. In the case of graphite we have a substance which increases its resistance even more than bismuth, and for which the variation of E with H is not so rapid as the field changes. The result of these conclusions is that the coefficient, R, instead of decreasing, is found to increase as H increases.

The method used in this experiment for getting E is based upon equation (14). If this equation is put into the form

$$E = \frac{i_x}{Hi_y}\left(\frac{X - X_1}{X_1}\right) \tag{19}$$

it is evident that the sign of E changes with the sign of $X - X_1$, and since this latter quantity determines the Hall coefficient we see that the Corbino constant and the Hall coefficient must change sign together. Chapman finds in the case of an alloy of bismuth and tin that as H increases one of these effects changes sign before the other. His measurements of the two effects were made separately, however, so we might expect some variations because of experimental errors. If for some reason a constant error in the value of R were introduced (as was obtained in the present experiment by shifting the position of the Hall terminals) then a result as obtained by Chapman might be expected—the curve for R would not cross the H axis at the same point as the curve for E. By the conditions of Chapman's experiment it is also possible for thermal effects in the measurement of R and E to be different, thus introducing an error in the comparison of the two effects.

As regards the disagreement of the experimental with the theoretical sign of the Hall and Corbino effects in zinc (and other metals) no perfectly satisfactory explanation has yet been given. According to J. J. Thomson[1] we might expect the value of H in equation (14) to have a sign different from that of the applied external magnetic field, especially where intramolecular fields are called into play by the magnetizing force.

With the observed values of R and E it is of interest to use equations (3) and (14) and calculate values of n and T on this theory of electron conduction, though in making these calculations one makes the assumption—possibly incorrect—that the magnetic field in which the electrons move is the same as that produced by the electromagnet of the experiment. As seen in the last columns of the above tables both the number of free electrons per unit volume and the mean free period of these electrons may be dependent upon the strength of the magnetic field. As has been previously pointed out,[2] one would expect T to be dependent upon H, but the reasons for variations in n are not so clear. In the case of graphite crystals it appears that n decreases with an increasing magnetic field, while for bismuth the increasing field produces an increasing n. Evidently the molecular structure of the metal is important in connection with this question. One may imagine a magnetic field influencing the stability of the orbits of electrons revolving in a molecule and thus, by varying the absorbing or emitting power of molecules for electrons, causing variations in n. Also, by causing molecular systems to orient themselves differently a magnetic field might change intra-molecular forces so as to affect the electronic radiation of molecules, and thus n would be changed. Our knowledge of these processes, however, is at present too vague to permit of any great specialization of the theory as described above.

SUMMARY.

1. The electron theory of metallic conduction has been used in developing a relationship between the three galvanomagnetic phenomena, —the Hall effect, the Corbino effect, and the change of resistance in a magnetic field.

2. It is found that the Hall coefficient divided by the specific resistance of the metal is equal to the Corbino constant.

3. A simple experiment has been devised so that all three of these galvanomagnetic effects may be determined in the same specimen with relatively simple operations.

4. In zinc and copper the Corbino constant is probably independent

[1] Corpuscular Theory of Matter.

[2] PHYS. REV., X., 4, p. 366, 1917.

of the magnetic field. In bismuth the three galvanomagnetic effects are found to behave in the usual manner but in crystalline graphite the resistance increases rapidly with the magnetic field, the Corbino constant decreases, and the Hall constant increases.

5. According to the usual interpretation of these experiments we may conclude that the number of free electrons per unit volume in a metal may be affected by placing the metal in a magnetic field. The free period is also affected. It appears, therefore, that both of these factors must be taken into account when one is considering the effect of a magnetic field on resistance.

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THE PHOTOLUMINESCENCE AND KATHODO-LUMINESCENCE OF CALCITE.

BY E. L. NICHOLS, H. L. HOWES AND D. T. WILBER.

THAT calcite is luminescent has been known for many years. E. Becquerel[1] mentioned it among the numerous substances whose phosphorescence of short duration was revealed by the use of his phosphoroscope. He gave the duration as half a second.

While Stokes[2] found calcite devoid of fluorescence, von Lommel[3] described all the specimens examined by him as showing a fine red glow in the path of a beam of sunlight transmitted through the crystal.

Sohncke[4] in his study of polarized fluorescence found spar from Iceland active, but crystals from Cumberland, England, inactive, a fact which doubtless explains the discrepancy between the observations of Stokes and of von Lommel.

G. C. Schmidt[5] confirmed the experiments of Sohncke and shared his view that the luminescence is not inherent in calcite but due to traces of foreign substances in solid solution.

Pochettino,[6] who more recently has examined many minerals under the action of kathode rays, finds calcites from some fourteen localities to possess phosphorescence of considerable duration and always with the same spectral distribution. It is however easy by a cursory examination of any considerable collection of calcites with the light of the iron spark, to confirm the statements of Sohncke and of Schmidt as to their varied behavior.

In a recent inspection by this means of the specimens available in the mineralogical laboratory of Cornell University we found many which showed the brief but vivid red glow typical of the calcites from Franklin Furnace, N. J.; others emitted a white phosphorescence, not brilliant but of long duration, an effect apparently common among stalagmites from various localities. A few pieces were pale green, a few amethystine

[1] E. Becquerel, La Lumiere, I., p. 354, 1867.
[2] Stokes, Phil. Trans., XI, p. 516, 1852.
[3] v. Lommel, Wiedemann's Annalen, XXI., p. 422, 1884.
[4] Sohncke, Wiedemann's Annalen, LVIII., p. 425, 1896.
[5] Schmidt, Wiedemann's Annalen, LX., p. 740, 1897.
[6] Pochettino, Il Nuovo Cimento (5), XVIII., p. 260, 1909.

in the color of their fluorescence, as though by the combination of a red and a blue band. One small crystal exhibited a strikingly brilliant and persistent blue phosphorescence, which came chiefly from a small portion, not as though a foreign particle were embedded there but rather as though some substance had gone into solution and filled a portion of the crystal bounded by growth planes which separated it from the remaining parts. Many of the calcites in this collection did not respond perceptibly to the light of the spark.

It is obvious that in the luminescence of calcite we have to do with phenomena as complicated as those found in the case of the phosphorescent sulphides and like the latter dependent upon the nature and amount of the dissolved material and upon treatment by heat. In contrast to the brilliant but brief phosphorescence noted by Becquerel, Headden[1] described calcites from Joplin, Mo., which after exposure to sunlight continued to glow with a reddish color for at least 14 hours and others from Fort Collins, Colo., the phosphorescence of which had a duration of several hours. The Joplin calcites were strongly thermoluminescent. Headden made a careful analysis of his material and found in addition to small amounts of Mn, Fe and Zn, measurable quantities of the rare earths. He was inclined to attribute the phosphorescent properties to the presence of yttrium.

Since a specimen of calcite from Lowville, N. Y., showed the reddish phosphorescence in the metamorphosed rock calcite and none in the "nail head" calcite-crystals—presumably recrystallized from solution—and since in general stalagmitic calcite did not show the red color, it was assumed that heat might be requisite to the production of this property. Accordingly a piece of calcite having a faint whitish glow was heated white hot and on cooling the calcined tip showed the red glow. To ascertain whether a chemical impurity was also necessary to produce the phosphorescence—as suggested by the work of Crookes[2] on the yellow phosphorescence of calcium oxide containing traces of yttrium—some of Kahlbaum's reprecipitated calcium carbonate was ignited but only the faintest white phosphorescence was obtained. A small amount of manganese chloride was added and the red phosphorescence showed at once. Some difficulty was experienced in reproducing this result until it was found hat a proper atmosphere is required during the ignition to prevent the manganese from turning black, presumably because of oxidation.

The best results were obtained by adding to one gram of calcium

[1] Headden, Am. Jour. of Science (4), XXI., p. 301, 1906.

[2] Crookes, Proc. Roy. Soc., 42, 115, 1887.

carbonate 1 c.c. of 1/50 molar manganese chloride solution, evaporating to dryness on a water bath, adding 50 mg. of oxalic acid crystals and grinding to a powder. The mixture was then ignited to a white heat. Ammonium chloride and sodium bicarbonate gave nearly as good results as oxalic acid, using the same weight.

The product obtained by this method glows with a reddish phosphorescence similar to that of the calcites when excited by ultra-violet light. It is, however, calcium oxide, the loss in weight being due to the loss of carbon dioxide. It deteriorates rapidly, ceasing to give any glow after standing a few weeks in the air, presumably due to the formation of calcium hydroxide.

This leads to the possibility that the phosphorescent centers are actually calcium oxide, with manganese or other oxide attached in a special manner produced by the heat treatment, and that the calcium carbonate in the calcite crystals is a matrix which holds these few scattered luminescent particles. It is conceivable that a metamorphosed calcite could be heated to a temperature at which the carbon dioxide would slowly be driven off, leaving a small per cent. of calcium oxide dissolved in the calcite.

A strong blue phosphorescence was obtained by using bismuth chloride as an activator in the place of the manganese chloride, and faint pinkish phosphorescence by employing antimony, copper, and uranium. These specimens were all strongly thermoluminescent.

Our interest in the luminescence of calcite originated in the desire to determine the law of decay of substances of brilliant but brief phosphorescence for comparison with that of the uranyl salts recently studied.[1]

A sample from Franklin Furnace appeared to suit our purpose. It showed the fine red after-glow characteristic of the calcites of that locality and although the duration of visibility (one half second), was about one hundred times that of the uranyl salts it was much too brief for examination by the method of single excitation. The fact that this calcite under the action of the kathode rays showed brilliant phosphorescence lasting several minutes added greatly to the interest of the comparison since, as has very recently been discovered by Misses Wick and McDowell,[2] certain uranyl salts at the temperature of liquid air likewise have phosphorescence of long duration when excited by the kathode discharge.

[1] Nichols and Howes, PHYS. REV. (2), IX., p. 292, 1917.

[2] Wick and McDowell, PHYS. REV. (2), XI., p. 421, 1918.

PHOTO-LUMINESCENCE

For the study of the brief photo-luminescence of the calcite a special phosphoroscope of the disk type was constructed, the chief novelty of which lay in the method of indicating the speed of revolution. It consists essentially of a drum *D* (Fig. 1) mounted upon a vertical shaft and driven by a small electric motor. It could be belted direct to the motor from any one of the three pulleys at the base of the shaft or, for low speeds, to a large pulley driven by worm gearing on the motor shafts. Thus the very great range of speeds necessary in phosphoroscopic measurements was readily obtained.

For the indication of speeds a brass disk was mounted on the shaft of the phosphoroscope below the drum. This passed between the poles of a little electromagnet *M*. Brushes *B*, *B* on the periphery of the disk and on the shaft *S* conveyed the current generated by this disk dynamo to the circuit of a d'Arsonval mirror galvanometer *G*. Adjustments of the resistance of the galvanometer circuit afforded deflections through the same range for the various speeds employed and enabled us to follow all changes in the rotation of the phosphoroscope regardless of the slip of the belt or of fluctuations in the current supply of the motor. Constancy of the field of the disk dynamo was secured by the use of storage cells and precautions were taken to eliminate thermal E.M.F.'s at the brushes. The phosphorescent material, in powdered form was mixed with white zapon varnish and applied with a brush to the outside of a brass collar *P* which fitted snugly over the slightly tapered drum of the phosphoroscope. A number of these collars, which were readily interchangeable, were provided to facilitate the examination of one substance after another.

Excitation was by means of an iron spark actuated by the convenient form of transformer devised for this purpose by Mr. W. S. Andrews,[1] or sometimes by means of a mercury arc in quartz, the tube of which was brought as close to the drum of the phosphoroscope as possible.

With the simple instrument just described exceedingly interesting and instructive observations of phosphorescence having a visible range between 1/100 seconds and 3 or 4 seconds may be made. The color-changes in the phosphorescence of bodies whose spectra have two or more bands which differ in wave-length and persistence are beautiful and most striking.

To determine the curve of decay a photometer of the type described in previous papers was employed. The eye piece *E* was usually directed to the rim of the drum 180° from the region of excitation (see Fig. 1), so that the spark would be hidden by the body of the drum.

[1] Andrews, General Electric Review, April, 1916.

A light filter *F* was interposed between the comparison lamp *C* and the Lummer-Brodhun cube. Before taking measurements, however, elaborate screening was employed to exclude stray light coming from the exciting source.

The usual method was to maintain a constant angle as above between points of excitation and of observations, varying the speed of the motor

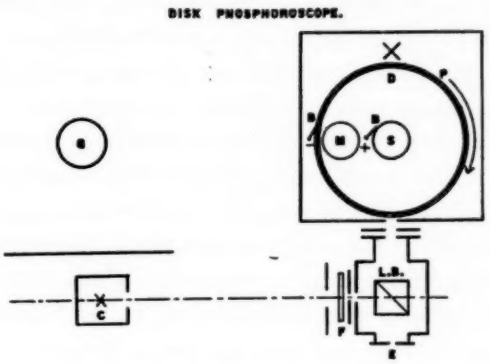

Fig. 1.

for successive settings. An assistant would adjust the motor speed, then move the comparison lamp along the bar until the observer announced a match. He would then immediately read the speed from the galvanometer scale which was mounted above the bar, and subsequently read the photometer setting. Thus the observer's eyes were spared, which is an important precaution in the readings at low speeds for which the brightness drops to less than 1/1,000 of the initial brightness. Occasional interchanges between observer and assistant in these experiments tended to eliminate the bias of both and check the results.

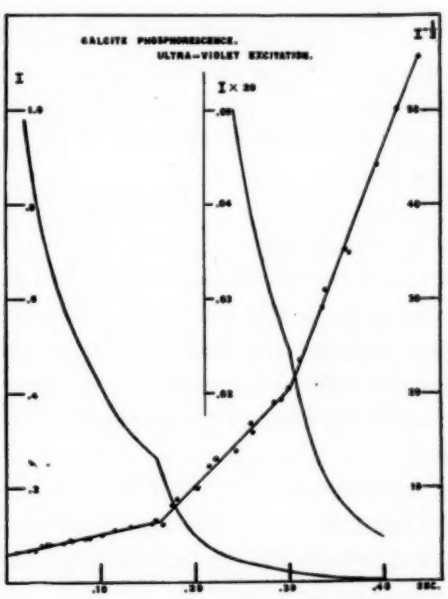

Fig. 2.

The curves thus obtained were of the form shown in Fig. 2. Plotted with the reciprocal of the square root of intensity as ordinates they indicate three

successive linear processes with increasing rates of decay, a type hitherto supposed to be peculiar to the phosphorescence of the uranyl salts.[1]

In the use of the disk photometer, as with any method in which periodically repeated exposures to excitation occur, results are comparable only throughout the range of speeds for which saturation is obtained. For the calcite under observation and with the intensity of excitation available, saturation was found to exist for all speeds of the disk less than that corresponding to .02 second decay. Any measurements in which the time interval from the close of excitation was not greater than .02 second were accordingly rejected.

Curves of the type shown in Fig. 2 differ so fundamentally from those universally associated with phosphorescence that one is at first inclined to question their validity and to ascribe their form to the methods necessarily employed in the study of phosphorescence of short duration. We went into this question very carefully in our investigation of the photophosphorescence of the uranyl salts and convinced ourselves by varying the form of apparatus, and the mode of excitation that the new curves were correct. In a recent paper on types of phosphorescence[2] we have indicated certain fundamental distinctions between the curves of decay of persistent phosphorescence and those characteristic of vanishing phosphorescence and have shown that the latter cannot be regarded as the first stages of curves of persistent phosphorescence.

There is indeed good reason to believe that we have in the two cases distinctly different effects. That both types can be produced in a single substance by varying the mode of excitation has been demonstrated for the uranyl salts, in the investigations already cited. It will be seen from the results to be given in the present paper that the same is true of calcite.

KATHODO-LUMINESCENCE.

To determine the law of decay of the phosphorescence produced by kathode rays a crystal *P* was placed in the bottom of a V-shaped vacuum tube, Fig. 3, and was viewed directly through the transmitting zone of the Lummer-Brodhun cube of the photometer. The comparison light for the reflecting zone was transmitted through a screen of ground glass or milk glass from a small tungsten lamp mounted upon a long photometer bar.

The approximate color match was obtained by the use of a suitable orange-yellow color screen. The excitation was produced by means of a large motor-driven influence machine.

[1] Nichols and Howes, *l. c.*

[2] Nichols and Howes, Proc. Nat. Acad. of Sci., 1918.

The time of close of excitation was automatically recorded on a chronograph; the subsequent times when the brightness of phosphorescence matched that of the comparison light placed at previously selected stations along the bar was indicated upon the same chronograph sheet by tapping a key in the chronograph circuit. The first observation was made as soon as practicable, *i. e.*, about .4 second after the close of excitation and readings were continued for about 300 seconds. The range of intensities was so great that the farther end of the bar was reached some time before the run was completed and the subsequent observations were obtained by substituting the milk glass screen for that of ground glass. The ratio of transmission of the two, which was about 1 : 80, was carefully measured.

Fig. 3.

The curve in Fig. 4 is characteristic of the kathodo-phosphorescence of these calcites, but as in the case of the curves of decay of the persistent phosphorescence of other substances the first two linear processes tend to merge, giving curves of the form shown in Fig. 5. This occurs, as

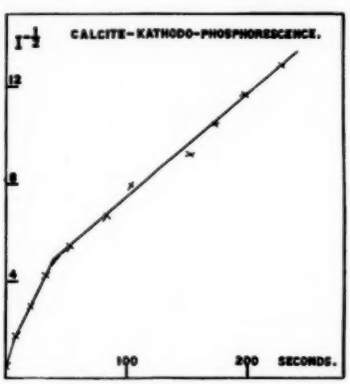

Fig. 4.

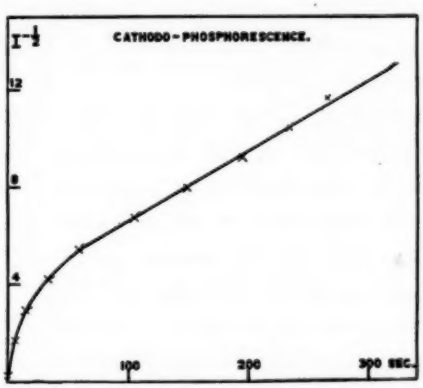

Fig. 5.

has been pointed out in a previous paper[1] whenever various portions of the substances are subjected to excitation of widely different intensity.

In photo-luminescence this is due to the penetration of the exciting light to the inner layers of a crystal;—in the case of kathodo bombardment it may also be due in part to the existence in the kathodo stream of particles having different velocities. It appeared that the colored film on the surface of calcite, due to prolonged exposure to kathode rays was conducive to the type of curve in Fig. 5 whereas the simpler form with well defined knees was characteristic of crystals with fresh unmodified surfaces.

[1] Nichols and Howes, *l. c.*, p. 301.

Shorter runs in which the stations along the bar were placed nearer together allowing just sufficient time to make the successive observations, served to establish more conclusively the linear character of the earlier processes. Fig. 6 gives the results of such a run. The initial observation in this and in the two preceding curves, is located at the origin of the time ordinate. Such readings were made before the close of excitation. The initial position is such as to indicate even more rapid decay of phosphorescence during the first half second than subsequently, but, by this method, it was impossible to obtain readings lying within this interval.

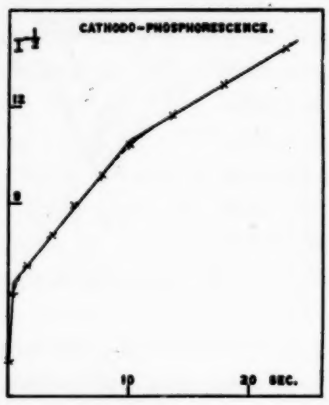

Fig. 6.

Since the whole of the visible processes of decay in the case of the *photo-phosphorescence* of these calcites occurs during this first half second of time it was deemed of interest to determine, if possible, what takes place during this interval when the excitation is by kathode bombardment. To this end we constructed a phosphoroscope capable of running in vacuo.

The disk, which was 4.2 cm. in diameter and which carried a flat periphery as in the instrument already described, was mounted at the upper end of vertical steel shaft 115 cm. in length. This passed coaxially through an iron tube 1.3 cm. in diameter. At the upper end this tube was threaded with a circular iron plate *N*, see Fig. 7, upon which a vacuum tube *V* with flanged lip was mounted.

The kathode *K* was in a side tube. It faced the periphery of the disk at a distance of about 0.5 cm. The iron plate served as anode.

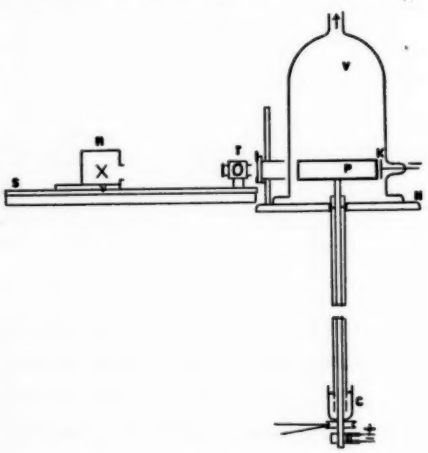

Fig. 7.

The iron tube within which the steel shaft revolved was provided with a deep iron cup *C* at the bottom, like the cistern of a barometer and the shaft passed through a mercury-tight stuffing box in the base of this cistern. The shaft was motor driven at various speeds from pulleys

attached below the stuffing box. When the cistern was filled with mercury and the vacuum tube exhausted with a Gaede pump, the mercury rose around the shaft, as in a barometer tube and a good kathode ray discharge was readily obtained and maintained. By means of countershafts and reduction gears, the speed of the disk could be varied from twenty or more revolutions a second to one revolution in one hundred seconds.

The intensity of the phosphorescence was measured by reading the position of the comparison lamp *R* on the bar *S* when the two fields were of equal intensity in photometer *T*. The range of the measurements made with this instrument is so great that they cannot adequately be represented in a single diagram. Indeed it was not found advantageous to make a detailed and continuous study of the entire range upon a single

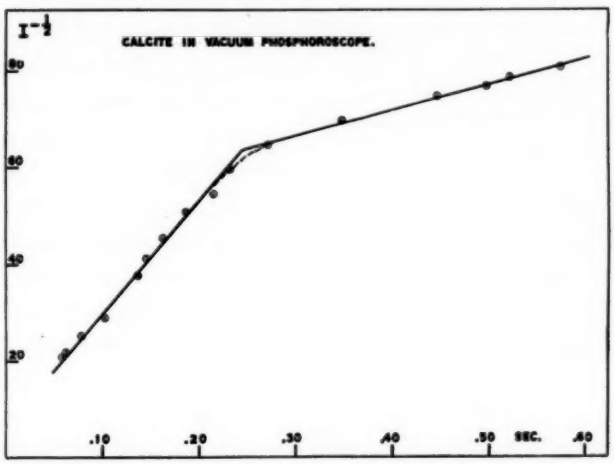

Fig. 8.

specimen because calcite, which is one of the most stable of luminescent substances under *photo-excitation*, becomes rapidly fatigued when exposed to kathode rays, with permanent discoloration of the surface layers.

Three sets of experiments were therefore made, using freshly prepared coatings of the powdered crystals:

1. *From .06 Second to .60 Second.*—A typical curve for this range is shown in Fig. 8. The whole of this curve lies within the time interval between the close of excitation and the first observation made by the method of single excitation already described (see Figs. 4, 5 and 6).

Like the curve for the vanishing phosphorescence produced by photo-excitation (Fig. 2) it is made up of sharply separated linear processes, but of the opposite type (*i. e.*, that usually associated with persistent phosphorescence).

It is clear therefore that so far as the law of decay is concerned the beginnings of persistent phosphorescence are *not* like the curves for vanishing phosphorescence. It is also evident that the *first process observable by the method of single excitation is not the initial process* in the case of persistent phosphorescence but is preceded by an earlier and more rapid decay lasting less than half a second.

2. *From .60 Second to 20 Seconds.*—This interval corresponds approximately to that covered by Fig. 9. Like the curves which had been obtained over this interval of time by the method of single excitation these resulting from measurements with the revolving disk in vacuo are characterized by a well-defined knee at about 10 seconds, separating two linear processes. The results of the two methods are identical as to type, as may be seen by comparing Fig. 9 with Fig. 6.

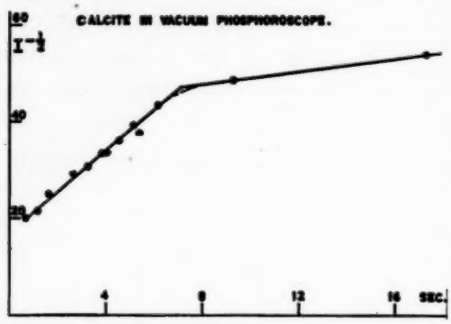

Fig. 9.

3. *From .6 Second to 300 Seconds.*—For this great range the difficulties of maintaining a constant vacuum and kathode discharge were such that we contented ourselves with establishing the existence of a knee in the curve at about 40 seconds corresponding to that obtained by the method of single excitation (Fig. 4), and of a linear process beyond.

The Effects of Temperature.

When a crystal of calcite from Franklin Furnace is heated certain changes in its photo-phosphorescence are noticeable. The after-glow, which is ruddy at + 20°, becomes reddish yellow and at higher temperatures a paler yellow. Before a red heat is reached phosphorescence becomes extinct, but the luminescent power returns on cooling. When excited at temperatures approaching that of liquid air the phosphorescence appears to be of a deeper, more intense, red than that produced at room temperature.

The temperatures at which the color changes noted above occur seem to correspond in general to those related to the production of thermo-luminescence in calcites from Joplin, Mo. Dr. Headden found that specimens from that locality began to glow at + 60°, were most brilliant at about + 180° and were not extinct at + 200°. We were able to confirm these observations in this regard in the case of a large crystal kindly furnished us by Professor Headden. Upon heating slowly one end of

the crystal in the Bunsen flame thermo-luminescence first appeared at the hot end, then receded gradually from the flame. The luminescence was confined to a rather narrow band which moved through the crystal with the conduction of heat, showing that the effect occurred between fairly well defined limits of temperature. The appearance after several minutes was as follows. The tip of the crystal, within the flame, appeared faintly red hot; then a dark non-luminescent zone below the red heat, but above that of thermo-luminescence, appeared; then the narrow thermo-luminescent belt,—very brilliant in the center and dying away more gradually toward the cooler end of the crystal. Our Franklin Furnace calcites, although strongly photo-luminescent, showed no trace of thermo-luminescence, and strangely enough, we could not excite to phosphorescence the crystals sent us by Professor Headden either by prolonged exposure to sunlight or by the action of the iron spark, although they were of the stock examined by him in 1906, at which time they exhibited the remarkably persistent phosphorescence after solar excitation described in his paper. The explanation of this loss in the phosphorescent properties of these calcites has not yet been found.

To determine more exactly the effects of temperature on the phosphorescence of our Franklin Furnace calcites the disk of the phosphoroscope used in the determination of photoluminescence was heated from below by means of a circle of minute gas jets *J*, *J*, Fig. 10—surrounding the shaft below the disk. The temperature of the ring on which the powdered calcite was mounted was measured by means of a Pt-Cu thermojunction *T* which could be inserted in a drill-hole in the metal of the ring.

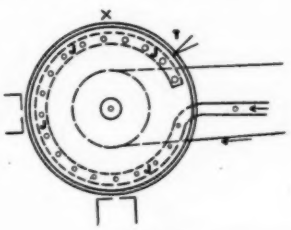

Fig. 10.

The phosphoroscope was driven at a constant speed and the brightness of the coating at various temperatures between + 20° C. and 350° was measured by means of a Lummer-Brodhun spectrophotometer, one collimator of which was directed towards the coating of the phosphorescent calcite. The other collimator was illuminated by the comparison lamp. Slit widths were maintained equal and constant and measurements were made by moving the comparison lamp along a photometer bar. Readings were made in the red of the spectrum (.65 μ) and in the green (.52 μ).

Accurate determinations by this method are difficult since they involve constant intensity of excitation and constant speed of the phosphoroscope throughout the run. Both conditions were hard to maintain with our

apparatus. Fair control of the speed could be only had by continual attention to the lubrication, particularly at the higher temperatures, where castor oil had to be substituted for the more volatile lubricants. This trouble could have been completely overcome by suitable reconstruction of the phosphoroscope but it seemed hardly worth while on account of the uncertain behavior of the iron spark. This source of excitation is altogether unsatisfactory where intense and really constant intensities are demanded. We have not as yet found an adequate substitute however for strong excitation by ultra-violet radiation.

At the higher temperatures the heated air greatly interfered with the action of the spark. To obviate this a quartz window was mounted between the disk and the spark, but we were greatly surprised to find that the excitation was reduced to a small fraction of its former intensity.

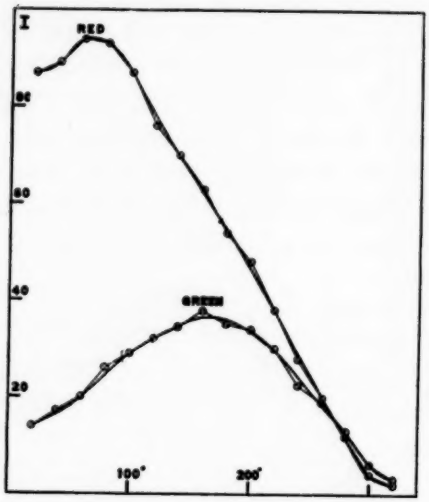

Fig. 11.

It appears that the luminescence of these calcites is chiefly produced by rays which are of shorter wavelength than those transmitted by quartz. This almost precluded the use of a mercury arc in quartz for these observations, although we did make several sets of spectrophotometric measurements with this source by way of a check.

The curves in Fig. 11 show the variations in the red and green throughout the range of temperatures mentioned above. Ordinates are intensities—referred to that of the corresponding region in the spectrum of the comparison light, and the readings are numerically comparable only if reduced to terms of energy. For the present purpose—which is to depict the relative effects of temperature, the curves in their present form will suffice. It will be noted that in both of these regions the brightness increases with the temperature. The red however reaches a maximum at about 70°, whereas the green continues to rise to about 160°. After passing the maximum the fall of both curves is continuous and rapid and they approach extinction;—*i. e.*, become too feeble for measurement, at about 350°.

The marked change in color is indicated by the fact that whereas if at 20° the region at .52 μ be made as bright as the corresponding region

in the spectrum of the comparison source, the red would be about 50 times as bright as the red of the comparison; while above 250° the relative brightnesses become nearly equal. At these higher temperatures, indeed, the distribution of intensities in the spectrum of the phosphorescence differs but little from that of a tungsten lamp up to about .5 μ. Its spectrum extends but little beyond that wave-length towards the violet, however, and the color is accordingly a fine luminous yellow.

SPECTROPHOTOMETRY OF THE PHOSPHORESCENCE AT + 20° AND + 200°.

The differing effects of temperature upon the two regions of the spectrum, as just described, are indicative of the presence of more than one band in the phosphorescence of these calcites. The components of the spectrum overlap, however, to such an extent as to be indistinguishable by spectroscopic inspection. We, undertook, therefore a detailed spectrophotometric study, making measurements every 50 Ångströms from .66 μ to .50 μ.

It was easier to ascertain the deviations from constancy of the phosphorescent surface, due to irregularities in the iron spark or fluctuations in the speed of the disk than it would have been to control these conditions throughout an entire spectrophotometric experiment. To this end a photometer was mounted so as to follow the fluctuations. The arrangement of the apparatus is indicated in Fig. 12.

The disk phosphoroscope D, already described, is driven by the motor M. The spark gap is within the enclosure S. The collimator A of the spectrophotometer H receives light from the phosphorescent ring on the disk, 180° from the point of excitation. The photometer P is so placed as to permit observations of the brightness of the ring at a point 90° beyond A. The comparison lamps C_1 and C_2, mounted on photometer bars as shown, serve respectively the collimator B and the photometer. The comparison lamps are similar 6-volt tungsten lamps in multiple on a circuit supplied by a suitable storage battery.

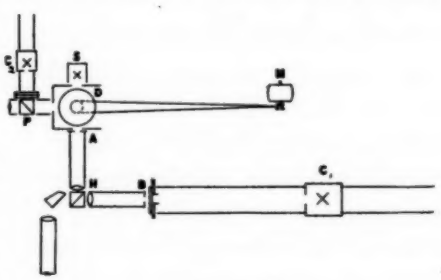

Fig. 12.

The method requires two observers—one at the eyepiece of the spectrophotometer, the other at the photometer. The latter follows the fluctuations of the phosphorescence and records its brightness at the instant when each spectrophotometric setting is made. It was thus possible

to eliminate, with a fair degree of accuracy, all errors due to changes in the excitation or speed.

Spectrophotometric measurements were made with this apparatus with the disk at + 20° and + 200°. The results are shown in the two curves in Fig. 13 in which ordinates are φ_λ/C where φ_λ is the intensity

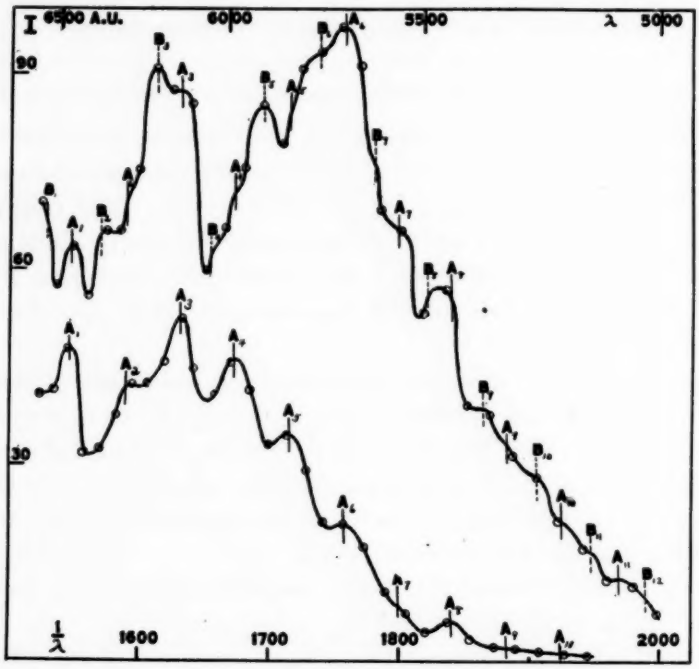

Fig. 13.

of the phosphorescence spectrum at wave-length λ and C is the intensity of the spectrum of the comparison lamp at the same wave-length. Abscissæ are frequencies, or $1/\mu \times 10^3$. The lower curve in the diagram is for 20° and the upper curve for 200°.

The curve for + 20° shows the phosphorescence spectrum to be complex, consisting of numerous overlapping components grouped about the main crest at $\lambda = 6131$ $(1/\lambda = 1631)$. If we assume that the components form a series with equal frequency differences, which is a common if not universal characteristic of luminescence spectra,[1] and take 42 units as the approximate interval, assigning one member of the series to the peak of the principal crest as above, we get the following series of components:

[1] Nichols and Merritt, PHYS. REV. (1), XXXIII., p. 354, 1911, and subsequent papers. Nichols, E. L., Proc. Am. Philos. Soc., LVI., p. 258, 1917.

TABLE I.

Series A. (*Interval 42.*)

Band.	λ.	$1/\lambda \times 10^3$.	Band.	λ.	$1/\lambda \times 10^3$.
A_1	.6464	1547	A_7	.5559	1799
A_2	.6293	1589	A_8	.5432	1841
A_3	.6131	1631	A_9	.5311	1883
A_4	.5977	1673	A_{10}	.5195	1925
A_5	.5831	1715	A_{11}	.5084	1967
A_6	.5692	1757			

TABLE II.

Series B. (*Interval 42.*)

Band.	λ.	$1/\lambda \times 10^3$.	Band.	λ.	$1/\lambda \times 10^3$.
B_1	.6545	1528	B_7	.5618	1780
B_2	.6369	1570	B_8	.5488	1822
B_3	.6203	1612	B_9	.5365	1864
B_4	.6046	1654	B_{10}	.5247	1906
B_5	.5896	1696	B_{11}	.5133	1948
B_6	.5754	1738	B_{12}	.5025	1990

These frequencies coincide in position with the subordinate crests that are visible as ripples in the curve in Fig. 13. Their positions are indicated by the short vertical lines marked *A* which cross the curve.

There are suggestions of a secondary or accompanying series, but the indications of it in the curve for 20° are not sufficient to establish its position or interval. In the curve for 200°, however, which is likewise plotted in Fig. 13, this secondary series (*B*) is strongly developed and becomes quite as prominent as series *A*. The strongest crest is now found at $\lambda = 5692$ ($1/\lambda = 1757$) and is broadened as if due to the presence of an unresolved doublet. Its edge towards the violet coincides in position with A_6 in the 20° curve. Its edge towards the red may be regarded as due to the outcropping of a member of series *B*,—the location and interval of which can be pretty definitely established from the crests B_3 and B_5. This series in both curves is indicated by short broken vertical lines (B_2, B_3, etc.). Its interval is also approximately 42 units of frequency.

The structure of the spectrum of this calcite resembles that of the phosphorescent sulphides which, as has been shown in a recent paper,[1] is made up of one or more series of overlapping components. The luminescence of this calcite however differs in at least one important respect from that of the sulphides. The light from the sulphides, as is

[1] Nichols, E. L., Proc. Am. Philos. Soc., LVI., p. 258, 1917.

well known is quenched by exposure to infra-red. The glow of the calcite is not appreciably affected by such radiation either during or after excitation. Of this we assured ourselves by careful observations.

To determine whether the Franklin Furnace calcite, which has been subjected to kathodo-bombardment acquires, even temporarily, the power of photo-phosphorescence a crystal was mounted in the V-shaped kathode ray tube already described (Fig. 3). A quartz lens was cemented over the open end of the tube, in place of the ground glass plug. The crystal glowed in the usual manner after excitation by the kathode rays.

Exposure to radiation from an iron spark mounted just above the quartz window produced the same brief, red after-glow, whether applied before, during, or after kathodo-excitation. The two effects appeared to be quite independent and could be simultaneously produced and superimposed.

It was in connection with these experiments that the following tentative observations on the effect of low temperatures were made.

The knee of the V-tube was submerged in liquid air and kathodo excitation was used. The luminescence seemed to the eye redder than at room temperature and very bright. The increase in intensity may, however, have been due to improvement in the vacuum due to cooling and a consequent change in the kathode discharge.

Photo-excitation gave the same brief ruddy afterglow as at room temperature, seemingly of a deeper red. No notable change in duration was observed. No quantitative measurements, of color, intensity or duration were made at these temperatures.

SUMMARY.

1. Samples of calcite from Franklin Furnace were subjected to excitation with the iron spark and to kathode bombardment.

2. The phosphorescence under photo-excitation is of the vanishing type, color ruddy, duration about .4 second.

3. Decay curves of the photo-phosphorescence are of the type characteristic of vanishing phosphorescence, previously recorded only in the case of certain uranyl salts. These curves show three "linear" processes.

4. The phosphorescence under kathodo-excitation is of the persistent type, red in color, measurable at 300 seconds after excitation.

5. Decay curves of kathodo-phosphorescence are of the more usual type, *e. g.*, that of the sulphides. These curves exhibit a series of linear processes.

6. The red and green regions of the spectrum are both affected by rising temperature, the red attaining a maximum intensity at about 70°

and the green at about 160°, while both become too dim to measure at about 350°.

7. An analysis of the spectrum reveals two series of overlapped bands, spaced equally in frequency units.

8. The Franklin Furnace calcites are not affected, as to the rate of decay of their phosphorescence, by the application of red or infra-red radiation.

9. Their characteristic ruddy phosphorescence appears to be due to the presence of traces of manganese. Calcium carbonate artificially prepared is rendered similarly phosphorescent by the addition of a manganese salt and subsequent heat treatment.

10. The action of kathode rays does not render these calcites even temporarily capable of persistent phosphorescence under photo-excitation.

11. The vanishing phosphorescence due to photo-excitation may be superimposed upon the persistent phosphorescence of kathodo-excitation either during or after exposure to the kathode rays.

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CORNELL UNIVERSITY,
June 19, 1918.

THE EVAPORATION OF SMALL SPHERES.

BY IRVING LANGMUIR.

THE evaporation of small spheres of iodine in quiet air has been studied by Harry W. Morse.[1]

The spheres of about one millimeter diameter were placed on the flat pan of a microbalance and weighed at intervals until they disappeared. The particles remained spherical in shape throughout the experiment.

The experiments indicated that the rate of loss of weight was accurately (within an average error of about one per cent.) proportional to the *radius* of the sphere and not to its *surface.*

This experimental fact was considered remarkable, but no theoretical explanation was suggested.

Several years ago I had occasion to make a rather detailed study of the convection of heat from small wires in air and other gases[2] and found that the heat loss by "convection" consists essentially of *conduction* through a film of relatively stationary gas around the wire. In other words, close to the surface of the wire the temperature gradient is so great that the heat carried by conduction is large compared to that carried by the motion, whereas at a greater distance the reverse is true. According to this theory the heat loss from a wire or other small body is given by the equation

$$w = s(\varphi_2 - \varphi_1), \tag{1}$$

where w is the heat loss in watts, s is the shape factor of the gas film, and $\varphi_2 - \varphi_1$ is equal to $\int_{T_1}^{T_2} k dT$ where k is the heat conductivity of the gas and T_2 and T_1 are the temperatures of the wire and the surrounding gas respectively.

This simple theory was found to agree excellently with experiment for wires of all sizes at temperatures varying from 100° up to the melting point of platinum or even that of tungsten and in several gases including air, nitrogen, argon, carbon dioxide, carbon monoxide, hydrogen and mercury vapor.

[1] Proc. Amer. Acad. Arts & Sciences, Vol. 45, April, 1910.

[2] Langmuir, PHYS. REV., *34*, 401, 1912. Proc. Amer. Inst. Elect. Eng., *31*, 1011, 1912. Trans. Amer. Electrochem. Soc., *23*, 299, 1913.

It is natural to assume that the evaporation of small objects in air should be closely related to the convection of heat from bodies of the same shape. By applying similar reasoning we are led to the equation

$$-\frac{dm}{dt} = s\int D d\rho. \tag{2}$$

Here $-dm/dt$ represents the rate of loss of weight, s is the shape factor of the relatively stationary gas film, D is the diffusion coefficient and ρ is the partial density of the vapor of the evaporating substance. For the low concentrations of vapors we are considering, D will be practically independent of ρ. From the ordinary gas laws we may place

$$\rho = \frac{pM}{RT}, \tag{3}$$

where p is the vapor pressure of the evaporating substance, M is the molecular weight, R the gas constant and T the absolute temperature.

The shape factor for a spherical shell is[1]

$$s = \frac{4\pi ab}{b-a}, \tag{4}$$

where b is the radius of the outside of the film of gas and a is the radius of the evaporating sphere. If we assume that b is very large compared to a we obtain

$$s = 4\pi a. \tag{5}$$

Substituting this together with (3) in (2) we find

$$-\frac{dm}{dt} = \frac{4\pi aDMp}{RT}. \tag{6}$$

This theory thus leads directly to the result that the rate of evaporation is proportional to the radius of the sphere.

It is of interest to calculate the value of D from Morse's experiments by means of the above equation. Morse found that

$$-\frac{dm}{dt} = ka. \tag{7}$$

The value of k can best be found from the experimental data by plotting $m^{2/3}$ against t. This gives a straight line whose slope is equal to $k/3\sqrt[3]{6/\pi\rho}$, where ρ is the density of the iodine spheres (4.95). In this way the values of k obtained by Morse are found to be

For first sphere, $k = 1.80 \times 10^{-6}$ g. per sec. per cm.

For second sphere, $k = 1.86 \times 10^{-6}$ g. per sec. per cm.

[1] See Flow of Heat through Furnace Walls. The Shape Factor, Langmuir, Trans. Amer. Electrochem. Soc., *22*, 55, 1913.

Combining (6) and (7)

$$k = \frac{4\pi DMp}{RT}. \tag{8}$$

Substituting $k = 1.83 \times 10^{-6}$; $M = 254$; $p = 271$ bars; $T = 298$; and $R = 83.1 \times 10^6$ we find

$$D = 0.053$$

for the diffusion coefficient of iodine vapor in air at 20° C.

Because free diffusion in all directions was hampered by the sphere resting on a flat surface it is probable that this value is somewhat too low, probably $D = 0.07$ would represent more nearly the actual diffusion coefficient.

As far as I know there are no available published data on the diffusion coefficient of iodine vapor in air, but the above value is a perfectly reasonable one when compared to other substances of high molecular weight. Thus the diffusion coefficient of carbon dioxide ($M = 44$) in air is 0.164; of acetic acid ($M = 60$) 0.122; of butyric acid ($M = 86$) 0.061; and of benzene ($M = 78$) 0.086. The iodine molecule is heavier but of smaller cross section than those of these substances, so that the diffusion coefficient 0.07 is in good agreement with the others.

These results therefore support the theory that the evaporation of the small spheres of iodine is determined simply by the rate of diffusion through the surrounding air and that no allowance needs to be made for air currents.

The results indicate also that the air in contact with the iodine surface is always saturated and that the actual rate of exchange of molecules between the solid iodine and its surrounding vapor is very rapid compared with the rate at which the vapor can diffuse away through the air.

This theory can undoubtedly be applied to evaporation from all very small objects. In the case of larger objects experimental data must be obtained as to the thickness of the film through which diffusion occurs just as has been done in the case of convection. In fact rough experiments seem to indicate that the thickness of these relatively stationary films are practically the same for diffusion phenomena as for convection.

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THE MOTION OF AN ELECTRICAL DOUBLET.

By Leigh Page.

IN a recent paper[1] it was shown that the expression obtained by Larmor for the reaction exerted on a moving mass by its own radiation is invalid because it is based on the assumption, that the effect of the damping which must accompany the emission of energy by the vibrators constituting the moving body, is negligible. The line of reasoning pursued was as follows. First the equation of motion of a single vibrating electron was derived rigorously from the electrodynamic equations, taking damping into account and including all mass reaction and radiation reaction terms of orders no higher than that of Larmor's expression, and was found to be of exactly the form, in so far as the velocity is concerned, demanded by the principle of relativity. Consequently it was concluded that the motion of a single vibrating electron is entirely in accord with this principle, and that such a particle is subject to no retardation opposed to its drift velocity. From this result it was inferred that a more complex body, such as a star, could not be subject to a reaction depending only upon its rate of radiation and its drift velocity. For a retardation which is a function of these two quantities alone must exist for all moving and radiating bodies, or for none. As it was shown rigorously to be non-existent for a single vibrating electron, it must be non-existent for a more complex radiator, such as a star. Hence Larmor's expression, which involves only the two quantities mentioned, must be invalid, and his assumption that the effect of damping can be neglected, unjustifiable.

A possible objection to this line of argument lies in the following consideration. In order that a single electron shall be accelerated and

[1] L. Page, Phys. Rev., 11, p. 376, 1918. In this paper the following errata should be noted:

Equation (12), p. 381. Denominator of second term within braces should contain the factor r.

Last equation, p. 384. For outstanding factor of left hand side read $-1/c$ instead of $-1/c^2$.

Last equation, p. 389. For coefficient of b^2 read 3/8 instead of 3/2.

Equation (29), p. 392. For coefficient of $(\partial\beta_0/\partial x)^2$ read $k^3(1+2\beta_0^2)$ in place of $k^3(1+2\beta_0)$.

Expression for x near bottom of page 392, and expressions for x and y near bottom of page 395. Second term should contain factor r.

hence radiate energy, a force must be exerted on it by some other charged particle in its vicinity. Now it might be contended, that while the equation of motion of the electron alone is entirely in accord with the principle of relativity, yet the moving doublet as a whole may suffer a retardation from the reaction of the radiation emitted. Hence it has seemed worth while to treat rigorously the problem of two charged particles, whose charges and masses are not necessarily equal, whose initial velocities are quite arbitrary, and whose subsequent motions are determined entirely by the fields produced by the two particles themselves. The analysis, which follows very simply from the equation of motion of the electron and the expansions in series of expressions for the electric and magnetic intensities which were published in the paper already referred to, includes all terms of orders no higher than the expression given by Larmor.

Let v_1 and v_2 be the velocities of the two particles, and r the line joining them. Pass a plane yz through r such that v_1 and v_2 have equal components normal to it. Then the particles may be considered as moving under the actions of each other in this plane, while the plane has a velocity $v_1 \cos\theta_1 = v_2 \cos\theta_2$ perpendicular to itself, θ_1 and θ_2 being the angles made by v_1 and v_2 respectively with the normal. Now the expression obtained by Larmor demands that the radiation emitted by the charged particles should exert a retardation on the doublet in the direction opposite to this normal velocity, whereas the principle of relativity denies the existence of any such force.

In determining the orders of the terms entering into the analysis, we shall suppose the charges on the two particles to be comparable in magnitude, and shall consider for the moment that they are rotating approximately in circles about some point in the moving plane (such as their center of gravity). Then equating the product of mass by acceleration to the external force on one of the particles we have

$$\frac{a}{r} \sim \beta^2$$

where $\beta \equiv v/c$, and a is the radius of the charged particle.

Hence if the force exerted by one of the particles on the other is taken to be of the zeroth order, inspection of the equation of motion (32) in the previous paper shows the terms to be of the following orders,

$$\frac{e^2 f}{6\pi a c^2} \text{ of zeroth order,}$$

$$\frac{e^2 f^2 \beta}{2\pi c^4} \text{ of fifth order,}$$

$$\frac{e^2\dot{f}}{6\pi c^3} \text{ of third order,}$$

$$\frac{e^2\ddot{f}a}{9\pi c^4} \text{ of sixth order,}$$

$$\frac{e^2\dddot{f}a^2}{18\pi c^5} \text{ of ninth order.}$$

As Larmor's expression is of the fifth order, only the first three terms in the equation of motion need be retained. As a matter of fact two more terms of this equation were evaluated than actually required to disprove Larmor's result. If we make use of the following notation,

$$k \equiv \frac{1}{\sqrt{1-\beta^2}}, \quad A \equiv \frac{e^2}{4\pi r^2}, \quad B \equiv \frac{e_1 e_2}{4\pi r^2}, \quad m_x \equiv \frac{x}{r}, \quad \gamma_x \equiv \frac{f_x r}{c^2}, \quad \delta_x \equiv \frac{\dot{f}_x r^2}{c^3},$$

where the subscript 1 refers to the particle a in the figure and the subscript 2 to b, the component (32) in the direction of motion of the reaction exerted on a by its own field may be written

$$K_{1_y} = A_1 k_1^3 \left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_y} + \tfrac{2}{3}k_1\delta_{1_y} + 2k_1^3\gamma_{1_y}^2\beta_1 \cdots \right]$$

and that (41) at right angles to the direction of motion

$$K_{1_n} = A_1 k_1^3 \left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_n}(1-\beta_1^2) + \tfrac{2}{3}k_1\delta_{1_n}(1-\beta_1^2) + 2k_1^3\gamma_{1_y}\gamma_{1_n}\beta_1(1-\beta_1^2)\cdots \right].$$

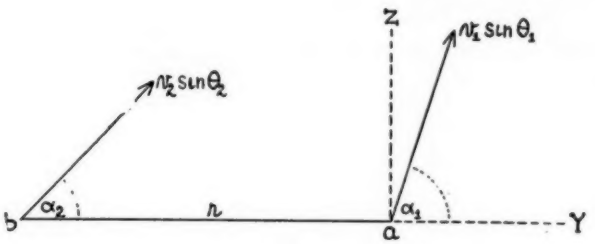

Fig. 1.

Hence if the x axis is taken perpendicular to the paper and upward a simple calculation gives

$$K_{1_x} = A_1 k_1^3 \beta^2 \cos^2\theta \tan\theta_1 \left[-\frac{2}{3}\frac{r}{a_1}(\gamma_{1_y}\cos\alpha_1 + \gamma_{1_n}\sin\alpha_1) + \tfrac{2}{3}(\delta_{1_y}\cos\alpha_1 + \delta_{1_n}\sin\alpha_1)\cdots \right] (1)$$

to the fifth order, and

$$K_{1_y} = A_1 k_1^3 \Big[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_y}(1 - \beta_1^2 + \beta_1^2 \sin^2\theta_1 \cos^2\alpha_1) - \frac{2}{3}\frac{r}{a_1}\gamma_1\beta_1^2 \sin^2\theta_1 \sin\alpha_1 \cos\alpha_1 + \tfrac{2}{3}\delta_{1_y}\cdots \Big] \quad (2)$$

$$K_{1_z} = A_1 k_1^3 \Big[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_z}(1 - \beta_1^2 + \beta_1^2 \sin^2\theta_1 \sin^2\alpha_1) - \frac{2}{3}\frac{r}{a_1}\gamma_{1_y}\beta_1^2 \sin^2\theta_1 \sin\alpha_1 \cos\alpha_1 + \tfrac{2}{3}\delta_{1_z}\cdots \Big] \quad (3)$$

to the third order, which is as far as required for the subsequent work.

From the expressions (23) and (24) of the previous paper we obtain for the force exerted by b on a

$$K_{1_x}' = e_1[\beta_1 \sin\theta_1(H_x \cos\alpha_1 - H_y \sin\alpha_1)]$$
$$= Bk_2^3\beta^2 \cos^2\theta \tan\theta_1 [\cos\alpha_1 ([1 - \beta_2^2 - \tfrac{2}{3}\beta_2^2 \sin^2\theta_2 \cos^2\alpha_2 - \gamma_{2_y} + \tfrac{2}{3}\delta_{2_y}) - \sin\alpha_1 (\tfrac{1}{2}\gamma_{2_z} - \tfrac{2}{3}\delta_{2_z})\cdots] \quad (4)$$

$$K_{1_y}' = e_1[E_y + \beta_1(H_x \sin\theta_1 \sin\alpha_1 - H_z \cos\theta_1)]$$
$$= Bk_2^3 [1 - \beta_2^2 - \tfrac{3}{2}\beta_2^2 \sin^2\theta_2 \cos^2\alpha_2 - \beta_1\beta_2(\sin\theta_1 \sin\theta_2 \sin\alpha_1 \sin\alpha_2 + \cos\theta_1 \cos\theta_2) - \gamma_{2_y} + \tfrac{2}{3}\delta_{2_y}\cdots], \quad (5)$$

$$K_{1_z}' = e_1[E_z + \beta_1(H_y \cos\theta_1 - H_x \sin\theta_1 \cos\alpha_1)]$$
$$= Bk_2^3 [\beta_1\beta_2(\sin\theta_1 \sin\theta_2 \cos\alpha_1 \sin\alpha_2) - \tfrac{1}{2}\gamma_{2_z} + \tfrac{2}{3}\delta_{2_z}\cdots]. \quad (6)$$

Now the total force on the particle a in the plane of vibration must be zero. Hence

$$K_{1_y}' + K_{1_y} = 0,$$
$$K_{1_z}' + K_{1_z} = 0.$$

Therefore,

$$A_1 k_1^3 \left(\frac{2}{3}\frac{r}{a_1}\gamma_{1_y} - \tfrac{2}{3}\delta_{1_y}\right)$$
$$= Bk_2^3 [1 - \beta_2^2 - \tfrac{3}{2}\beta_2^2 \sin^2\theta_2 \cos^2\alpha_2 + \beta_1^2 - \beta_1^2 \sin^2\theta_1 \cos^2\alpha_1 - \beta_1\beta_2 (\sin\theta_1 \sin\theta_2 \sin\alpha_1 \sin\alpha_2 + \cos\theta_1 \cos\theta_2) - \gamma_{2_y} + \tfrac{2}{3}\delta_{2_y}\cdots],$$

$$A_1 k_1^3 \left(\frac{2}{3}\frac{r}{a_1}\gamma_{1_z} - \tfrac{2}{3}\delta_{1_z}\right) = Bk_2^3 [\beta_1\beta_2 \sin\theta_1 \sin\theta_2 \cos\alpha_1 \sin\alpha_2 - \beta_1^2 \sin^2\theta_1 \sin\alpha_1 \cos\alpha_1 - \tfrac{1}{2}\gamma_{2_z} + \tfrac{2}{3}\delta_{2_z}\cdots].$$

Remembering that $\beta_1 \cos\theta_1 = \beta_2 \cos\theta_2$,

$$A_1 k_1^3 \left[\frac{2}{3}\frac{r}{a_1}(\gamma_{1_y}\cos\alpha_1 + \gamma_{1_z}\sin\alpha_1) - \tfrac{2}{3}(\delta_{1_y}\cos\alpha_1 + \delta_{1_z}\sin\alpha_1) \right]$$

$$= Bk_2^3 [\cos\alpha_1(1 - \beta_2^2 - \tfrac{3}{2}\beta_2^2 \sin^2\theta_2 \cos^2\alpha_2 - \gamma_{2_y} + \tfrac{2}{3}\delta_{2_y})$$

$$- \sin\alpha_1(\tfrac{1}{2}\gamma_{2_z} - \tfrac{2}{3}\delta_{2_z})],$$

from which

$$K_{1_x}' = A_1 k_1^3 \beta^2 \cos^2\theta \tan\theta_1 \left[\frac{2}{3}\frac{r}{a_1}(\gamma_{1_y}\cos\alpha_1 + \gamma_{1_z}\sin\alpha_1) \right.$$

$$\left. - \tfrac{2}{3}(\delta_{1_y}\cos\alpha_1 + \delta_{1_z}\sin\alpha_1)\cdots \right], \quad (7)$$

whence

$$K_{1_x}' + K_{1_x} = 0.$$

The resultant force normal to the plane of vibration is zero for each particle, and hence zero for the doublet. In fact the analysis shows that the retardation K_x on one of the particles due to the reaction of its own field is exactly annulled by the accelerating force K_x' produced by the magnetic field of the other particle.

In order to illustrate the fact that a correct solution of the problem is impossible unless damping is taken into account, consider two particles which have equal radii and equal and opposite charges. At the instant considered let them have velocities of which the components in the plane of vibration are equal and opposite, and at right angles to the line joining them. As before, the plane of their motion will be supposed to have a velocity $v\cos\theta$ along the normal. Then

$$A_1 = -B,\ \beta_1 = \beta_2,\ \gamma_1 = -\gamma_2 \equiv \gamma,\ \delta_1 = -\delta_2 \equiv \delta,\ \alpha_1 = \alpha_2 - \pi = \pi/2.$$

Hence

$$K_{1_x} = Ak^3\beta^2 \sin\theta\cos\theta \left[-\frac{2}{3}\frac{r}{a}\gamma_z + \tfrac{2}{3}\delta_z\cdots \right],$$

$$K_{1_y} = Ak^3 \left[-\frac{2}{3}\frac{r}{a}\gamma_y(1 - \beta^2) + \tfrac{2}{3}\delta_y\cdots \right],$$

$$K_{1_z} = Ak^3 \left[-\frac{2}{3}\frac{r}{a}\gamma_z(1 - \beta^2\cos^2\theta) + \tfrac{2}{3}\delta_z\cdots \right],$$

$$K_{1_x}' = -Ak^3\beta^2 \sin\theta\cos\theta[\tfrac{1}{2}\gamma_z - \tfrac{2}{3}\delta_z\cdots],$$

$$K_{1_y}' = -Ak^3[1 - 2\beta^2\cos^2\theta + \gamma_y - \tfrac{2}{3}\delta_y\cdots],$$

$$K_{1_z}' = -Ak^3[\tfrac{1}{2}\gamma_z - \tfrac{2}{3}\delta_z\cdots].$$

Therefore

$$K_{1_x}' + K_{1_x} = -Ak^3\beta^2 \sin\theta\cos\theta\left[\frac{2}{3}\frac{r}{a}\gamma_x + \tfrac{1}{2}\gamma_x - \tfrac{4}{3}\delta_x \cdots\right] \quad (8)$$

is the retarding force on one of the particles in the direction perpendicular to the plane of the rotation.

Suppose we neglect damping. Then the particles will move in a circle about a point half way between them, and

$$\gamma_y = -2\beta^2 \sin^2\theta, \qquad \gamma_x = 0, \qquad \delta_y = 0, \qquad \delta_x = -4\beta^3 \sin^3\theta,$$

whence

$$K_{1_x}' + K_{1_x} = -Ak^3\beta^2 \sin\theta\cos\theta[\tfrac{16}{3}\beta^3 \sin^3\theta]$$

$$= -\frac{4e^2}{3\pi r^2}\beta^5 \sin^4\theta\cos\theta$$

$$= -\frac{e^2f^2}{3\pi c^4}\beta\cos\theta,$$

and for the two particles the retarding force is

$$K_x = -\frac{2e^2f^2}{3\pi c^4}\beta\cos\theta.$$

Now the rate of radiation from the doublet is

$$R = \frac{2e^2f^2}{3\pi c^3}.$$

Therefore,

$$K_x = -\frac{1}{c^2}Rv_x,$$

which is just Larmor's expression for the retardation.

But in order to satisfy the equation of motion in the plane of rotation,

$$K_{1_y}' + K_{1_y} = 0,$$

$$K_{1_x}' + K_{1_x} = 0,$$

or

$$\gamma_y = -\frac{3}{2}\frac{a}{r}(1 - \beta_2),$$

$$\gamma_x = -\frac{3}{2}\frac{a}{r}(-\tfrac{4}{3}\delta_x),$$

showing that undamped motion is inconsistent with the electrodynamic equations. Substituting this value of γ_x in (8) we see that the total force normal to the plane of vibration is zero for each charged particle individually, and hence for the doublet as a whole.

It may be of interest to discuss the case where the mass of one of the particles is not entirely electromagnetic. For instance, consider a single electron and a positive nucleus rotating about their center of gravity. The experiments of Bucherer, Neumann, and others have proved that the electron's mass is entirely electromagnetic, but that of the nucleus may be in part mechanical, *i. e.*, a constant, independent of the velocity. The problem is then no longer purely a matter of electrodynamics, and the close connection between the relativity transformations and the electrodynamic equations can not be advanced as showing that there can be no retardation from the reaction of the radiation. If the electron is at a in the figure and the nucleus at b, and if the velocity components in the plane of vibration are taken opposite and perpendicular to the line joining the particles, and of such a magnitude as to make the motion approximately circular, we have for the electron

$$K_{1_x} = A_1 k_1^3 \beta^2 \cos^2\theta \tan\theta_1 \left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_x} + \tfrac{2}{3}\delta_{1_x}\cdots \right],$$

$$K_{1_y} = A_1 k_1^3 \left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_y}(1-\beta_1^2) + \tfrac{2}{3}\delta_{1_y}\cdots \right],$$

$$K_{1_z} = A_1 k_1^3 \left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_z}(1-\beta_1^2\cos^2\theta_1) + \tfrac{2}{3}\delta_{1_z}\cdots \right],$$

$$K_{1_x}' = -Bk_2^3\beta^2\cos^2\theta\tan\theta_1\left[-\tfrac{1}{2}\gamma_{2_x}+\tfrac{2}{3}\delta_{2_x}\cdots\right],$$

$$K_{1_y}' = -Bk_2^3\left[1-\beta_2^2+\beta_1\beta_2(\sin\theta_1\sin\theta_2-\cos\theta_1\cos\theta_2)-\gamma_{2_y}+\tfrac{2}{3}\delta_{2_y}\cdots\right],$$

$$K_{1_z}' = -Bk_2^3\left[-\tfrac{1}{2}\gamma_{2_z}+\tfrac{2}{3}\delta_{2_z}\cdots\right],$$

and for the nucleus

$$K_{2_x} = A_2 k_2^3 \beta^2 \cos^2\theta \tan\theta_2 \left[\frac{2}{3}\frac{r}{a_2}\gamma_{2_x} - \tfrac{2}{3}\delta_{2_x}\cdots \right],$$

$$K_{2_y} = -M_2\gamma_{2_y} + A_2 k_2^3 \left[-\frac{2}{3}\frac{r}{a_2}\gamma_{2_y}(1-\beta_2^2) + \tfrac{2}{3}\delta_{2_y}\cdots \right],$$

$$K_{2_z} = -M_2\gamma_{2_z} + A_2 k_2^3 \left[-\frac{2}{3}\frac{r}{a_2}\gamma_{2_z}(1-\beta^2\cos^2\theta_2) + \tfrac{2}{3}\delta_{2_z}\cdots \right],$$

$$K_{2_x}' = -Bk_1^3\beta^2\cos^2\theta\tan\theta_2\left[\tfrac{1}{2}\gamma_{1_x}-\tfrac{2}{3}\delta_{1_x}\cdots\right],$$

$$K_{2_y}' = -Bk_1^3\left[-1+\beta_1^2-\beta_1\beta_2(\sin\theta_1\sin\theta_2-\cos\theta_1\cos\theta_2)-\gamma_{1_y}+\tfrac{2}{3}\delta_{1_y}\cdots\right],$$

$$K_{2_z}' = -Bk_1^3\left[-\tfrac{1}{2}\gamma_{1_z}+\tfrac{2}{3}\delta_{1_z}\cdots\right],$$

where

$$M_2 \equiv \frac{c^2}{r}m_2,$$

and m_2 is the mechanical mass of the nucleus.

Put

$$k_2^2 M_2 + \tfrac{2}{3} A_2 k_2^3 \frac{r}{a_2} = \tfrac{2}{3} A_2 k_2^3 \frac{r}{b_2}.$$

Then, as the force on each particle in the plane of the vibration must vanish,

$$A_1 k_1^3 \left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_y} + \tfrac{2}{3}\delta_{1_y} \right]$$
$$= Bk_2^3 [1 - \beta_2^2 + \beta_1^2 - \beta_2^2 \cos^2\theta_2 + \beta_1\beta_2 \sin\theta_1 \sin\theta_2 - \gamma_{2_y} + \tfrac{2}{3}\delta_{2_y}],$$

$$A_1 k_1^3 \left[-\frac{2}{3}\frac{r}{a_1}\gamma_{1_z} + \tfrac{2}{3}\delta_{1_z} \right] = Bk_2^3 [-\tfrac{1}{2}\gamma_{2_z} + \tfrac{2}{3}\delta_{2_z}],$$

$$A_2 k_2^3 \left[-\frac{2}{3}\frac{r}{b_2}\gamma_{2_z} + \tfrac{2}{3}\delta_{2_y} \right]$$
$$= Bk_1^3 [-1 + \beta_1^2 - \beta_2^2 + \beta_1^2 \cos^2\theta_1 - \beta_1\beta_2 \sin\theta_1 \sin\theta_2 - \gamma_{1_y} + \tfrac{2}{3}\delta_{1_y}].$$

$$A_2 k_2^3 \left[-\frac{2}{3}\frac{r}{b_2}\gamma_{2_z} + \tfrac{2}{3}\delta_{2_z} \right] = Bk_1^3 [-\tfrac{1}{2}\gamma_{1_z} + \tfrac{2}{3}\delta_{1_z}].$$

So, for the electron

$$K_{1_x}' + K_{1_x} = 0$$

as before. For the nucleus, however,

$$K_{2_x}' + K_{2_x} = -A_2 k_2^3 \beta^2 \cos^2\theta \tan\theta_2 \tfrac{2}{3} \gamma_{2_x}\left(\frac{r}{b_2} - \frac{r}{a_2}\right)$$
$$= -\beta^2 \cos^2\theta \tan\theta_2 k_2^2 M_2 \gamma_{2_x}. \quad (9)$$

Let us take the charge on the nucleus equal to that on the electron. Then $A_2 = B$, and

$$\gamma_{2_x} = \frac{b_2}{r}(\delta_{2_x} - \delta_{1_x})$$
$$= \frac{b_2}{r}\left(\frac{\gamma_{2_y}^2}{\beta_2 \sin\theta_2} + \frac{\gamma_{1_y}^2}{\beta_1 \sin\theta_1}\right).$$

But, for approximately circular motion

$$\frac{\gamma_{2_y}}{b_2} = \frac{\gamma_{1_y}}{a_1}, \qquad \frac{\beta_2 \sin\theta_2}{b_2} = \frac{\beta_1 \sin\theta_1}{a_1}.$$

Hence

$$K_x = -\beta \cos\theta\, b_2 m_2 \frac{f_{2_y}^2}{c^2}\left(1 + \frac{a_1}{b_2}\right),$$

or, if we denote the electromagnetic mass of the electron by m_1' and

that of the nucleus by m_2'

$$K_x = -\frac{\beta}{c}\cos\theta \cdot \frac{e^2 f_{2_y}^2}{6\pi c^3} \frac{\left(1 + \frac{m_2 + m_2'}{m_1'}\right)}{\left(1 + \frac{m_2'}{m_2}\right)},$$

Now the rate of radiation from the doublet is

$$R = \frac{e^2}{6\pi c^3}(f_{2_y}^2 + 2f_{1_y}f_{2_y} + f_{1_y}^2)$$

$$= \frac{e^2 f_{2_y}^2}{6\pi c^3}\left(1 + \frac{m_2 + m_2'}{m_1'}\right)^2.$$

Therefore

$$K_x = -\frac{1}{c^2} Rv_x \frac{1}{\left(1 + \frac{m_2'}{m_2}\right)\left(1 + \frac{m_2 + m_2'}{m_1'}\right)}. \qquad (10)$$

This expression vanishes if the mass of the nucleus is entirely electromagnetic. If, however, its mass is assumed to be practically all mechanical, and eighteen hundred times the electromagnetic mass of the electron,

$$K_x = -\frac{1}{1800}\frac{1}{c^2} Rv_x,$$

which is a small fraction of Larmor's expression. In fact the above analysis shows that his result would be true only if the entire mass of the vibrating system were mechanical. If its mass is entirely electromagnetic, as has been assumed in this and the preceding paper, there is no retardation, and the principle of relativity is confirmed. For a doublet consisting of an electromagnetic electron rotating about a mechanical nucleus there is a retardation, which, however, is a very small part of that calculated by Larmor.

SUMMARY.

(*a*) The problem of two charged particles with electromagnetic masses, moving under the action of each other's fields in a plane which has a drift velocity along its normal, has been treated rigorously, and no retardation found to exist as a result of the radiation emitted.

(*b*) Larmor's expression for the retardation due to the reaction of the radiation from the moving doublet has been shown to be a consequence of his failure to take into account the effect of damping. The assumption that damping is negligible leads to a contradiction with the electrodynamic equations. When damping is taken into consideration this inconsistency disappears, and the retarding force vanishes.

(*c*) Damping may be neglected, and hence Larmor's expression holds, only when the mass of the vibrating system is assumed to be entirely mechanical (not a function of the velocity). If the radiating doublet consists of an electron of electromagnetic mass and a positive nucleus of a mechanical mass eighteen hundred times as great, there is a retardation amounting to one eighteen hundredth part of that calculated by Larmor.

SLOANE PHYSICS LABORATORY,
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June 1, 1918.

LAW OF MOTION OF A DROPLET MOVING WITH VARIABLE VELOCITY IN AIR.

BY RAYMOND B. ABBOTT.

THE object of this research was to find the law of motion of a droplet moving with variable velocity in a viscous medium, air; and also to find whether the method could be used to determine accurately the value of the electric charge given to the droplet, while falling between charged condenser plates.

Stokes' law[1] has been hitherto supposed to hold only for uniform motion in a straight line. Its limitation is given by Rayleigh[2] to be such that the radius r should be small compared to $\mu/\rho V$, where μ is the coefficient of viscosity of air, ρ is the density of air, and V is the velocity of the droplet. Many of the speeds observed in the experiments herein described are well within this limitation, but the motion is neither uniform nor in a straight line; for the droplet falls through a horizontal electric field, the charge on condenser plates alternating sixty times per second and varying according to the sine law. The droplet therefore oscillates with a horizontal motion while it falls.

The method used was a modified form of the one suggested by E. P. Lewis and used by W. A. Shewhart.[3] The original idea, evidently, was inspired by the work of Millikan on the "Elementary Electrical Charge."[4]

In general, the present method is to photograph the path of the particle as it falls through a periodically changing electric field and to find the law of motion from the form of the curve.

Shewhart showed conclusively that the amplitude of oscillation of the droplet is here proportional to the maximum electric force upon it. But the increase of wave-length which he found, where the field intensity was increased, can be explained by the fact that convection currents are set up in the air where more intense ionization takes place from the sharp edges of the condenser plates, as they were arranged.

He assumed that the path of the droplet was approximately a sine curve but did not find its exact form. In the present work enlargements

[1] G. G. Stokes, Mathematical and Physical Papers, Vol. III., p. 59.

[2] Phil. Magazine, Vol. 36, p. 365, 1893.

[3] PHYS. REV., May, 1917.

[4] PHYS. REV., August, 1913.

of direct photographs of path were made, and the form of the curve was compared with an accurately constructed sine curve. Also, data were obtained showing the angle of lag of the droplet behind the field intensity. Further, the law of motion of spheres of small density and known mass, falling in air, was experimentally determined. Shewhart's apparatus was used, with a feature added, which showed on the photograph the exact position of the droplet at some known value of the field intensity.

Two methods were used for doing this; first, a synchronous motor with a narrow vane attached to the shaft cut off the beam of light[1] when the field intensity was zero in value. This showed as a short break in the photographed path of the droplet (see Fig. 2). A disk of insulating material provided with a metallic ring and two contact makers 180 degrees apart, placed on the shaft of the motor, provided a means for finding the position of the shaft for zero electromotive force of the power circuit. Two brushes connected in series to the power circuit and a voltmeter, shunted with a condenser, provided a circuit which was closed whenever one of the contact makers on the disk hit a brush. By rotating the brushes a position could be found for any desired value of the field intensity.

The apparatus for the second method consisted of a spark gap connected across a large condenser which was charged by the power circuit. The spark occurred twice during each cycle and illuminated the falling droplet, causing a dark spot on the photographed path of the particle. The time of the spark with reference to the zero value of the electromotive force was measured by noting the position of the vane on the shaft of the synchronous motor when the spark occurred. This was done in a dark room where the rotating vane could be seen only when the spark illuminated it. With the apparatus used, the spark occurred approximately five degrees after the zero value of the field intensity was reached (see Fig. 3).

HORIZONTAL AND VERTICAL COMPONENTS OF AIR RESISTANCE.

Fig. 1 shows a direct enlargement of the photographed path of a droplet. The points of a sine curve, mathematically constructed upon it, show that the path is a sine curve. The electromotive force which charges the two condenser plates follows the sine law very closely. This is known from oscillographic analyses made in the departments of physics and electrical engineering. The spot of light shows where the droplet is located with reference to the electromotive force of the condenser plates.

One can now definitely express displacement, velocity and acceleration

[1] See Shewhart, PHYS. REV., May, 1918, p. 427.

FIG. 1.

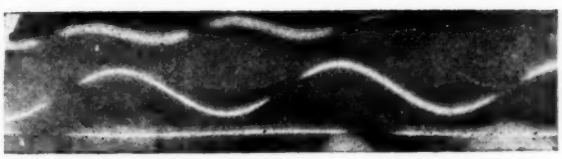

FIG. 2.

FIG. 3.

RAYMOND B. ABBOTT.

in the horizontal direction in terms of the sine law and an unknown function $f(y)$ representing the air resistance; and from the equation of motion, the form of the function $f(y)$ can be ascertained. This will be the first step taken in our effort to discover the law of air resistance in this case.

Accelerated motion of a body in a viscous medium is opposed by two kinds of resistance. There is a viscous or frictional resistance, and also an inertia resistance due to an apparent increase in the mass of the moving body, known as a "fictitious mass." The viscous resistance always opposes motion, at constant or at variable speed, while an inertia resistance opposes only a change of speed.

Let the maximum force on the droplet due to the charge (Ne) and the field intensity X be

$$A = (Ne)X. \tag{1}$$

Let the angle corresponding to the distance fallen by the droplet be x, and the angle of lead of the electromotive force ahead of the droplet be γ; then, since the electromotive force follows the sine law, the electric force on the droplet will be

$$F(y) = A \sin (x + \gamma). \tag{2}$$

The angle $(x + \gamma)$ must increase at a constant rate in order to satisfy the conditions governing the generation of the electromotive force at the dynamo. Two cases are open for investigation, involving the variation of the angle x of the droplet. The angle x either increases at a constant rate and γ is constant, or x may vary in such a manner that the average increase in its value for each quarter cycle is the same for every quarter; then γ must vary in such a manner as will make $(x + \gamma)$ increase at a constant rate at all times. The first case will now be considered, in which γ and dx/dt are constant.

Since the path of the droplet is known from Fig. (1) to be a sine curve, the following quantities can be expressed in terms of the sine law:

$$\begin{gathered} \text{Displacement } y = a \sin x \\ \text{Angular velocity } \omega = dx/dt \\ \text{Horizontal velocity } V_y = a\omega \cos x \\ \text{Horizontal acceleration } p_y = -a\omega^2 \sin x. \end{gathered} \tag{3}$$

If $f(y)$ represents the expression for the resisting force parallel to Y, and M the mass of the droplet, the completed equation of motion parallel to the Y axis is

$$A \sin (x + \gamma) - f(y) = M(-a\omega^2 \sin x). \tag{4}$$

Solve for $f(y)$, expand $\sin(x+\gamma)$ and collect coefficients of $\sin x$ and $\cos x$; the result is

$$f(y) = (A\cos\gamma + Ma\omega^2)\sin x + (A\sin\gamma)\cos x. \tag{5}$$

Let $\alpha = (A\cos\gamma + Ma\omega^2)$, $\beta = (A\sin\gamma)$ and $c = \sqrt{\alpha^2+\beta^2}$, $\tan j = \alpha/\beta$.

The function $f(y)$ written in terms of these constants is:

$$f(y) = c\cos(x-j). \tag{6}$$

The equation of motion is, therefore,

$$A\sin(x+\gamma) - c\cos(x-j) = M(-a\omega^2\sin x). \tag{7}$$

Equation (5) or (6) may be written $f(y) = \alpha\sin x + \beta\cos x$ which expresses both kinds of resistance, due to viscosity and inertia; for $\alpha\sin x$ is in phase with the acceleration, and $\alpha\cos x$ is in phase with the velocity. Further, $\beta\cos x$ must contain all of the viscous resistance; for the latter must be at its maximum value when the velocity is at its maximum value, but $\alpha\sin x$ is zero at this time.

In regard to the absolute value of γ, the value of $\cos\gamma$ may be such for every droplet that $a\omega^2M + A\cos\gamma = 0$, that is, γ greater than $\pi/2$ and $M - (A\cos\gamma) = 0$. If α does not equal zero, it contains a fictitious mass term, which for air, may be small enough to neglect. The fictitious mass of a sphere moving in a perfect fluid is discussed in Lamb's Hydrodynamics.[1] It is shown that the increase in the inertia of the sphere is one half the mass of the fluid displaced. In the case here considered the fluid is air which is not a perfect fluid, but the relation gives an idea of the magnitude of the term which appears as an additional inertia. The theoretical value would be approximately one half the ratio of the density of air to the density of water multiplied by the mass of the water droplet.

$$\text{Fictitious mass} = \frac{.00119}{2}M = .0006M.$$

The inertia resistance expressed in terms of the acceleration is

$$\alpha\sin x = \left(\frac{a\omega^2}{a\omega^2}\right)\alpha\sin x = \frac{\alpha}{a\omega^2}(a\omega^2\sin x) = \beta_2 p_y, \tag{7a}$$

where p_y is the horizontal acceleration, and

$$\beta_2 = \frac{-\alpha}{a\omega^2}.$$

This gives β_2 as the fictitious mass term, referred to above.

[1] Lamb, p. 130.

The viscous resisting force, expressed in terms of the velocity, is

$$(7b) \qquad \beta \cos x = \left(\frac{a\omega}{a\omega}\right)\beta \cos x = \frac{\beta}{a\omega}(a\omega \cos x) = \beta_y V_y.$$

This indicates that the resisting force is proportional to the velocity; and this is what one would expect, because most of the velocities encountered here are within the range of Stokes' law. With these conditions in mind, one can write the equation of motion in the horizontal direction in a different form from (7):

$$(8) \qquad A \sin (x + \gamma) - \beta_y a\omega \cos x = (M + \beta_2)(-a\omega^2 \sin x);$$

and in general,

$$(9) \qquad F(y) - \beta_y V_y = (M + \beta_2)p_y.$$

It will now be assumed that a similar form of equation holds for motion in the vertical direction; for experimental results have been found to verify the validity of the assumption.

V_x = vertical velocity at time t,

V_x' = terminal velocity,

m = mass of droplet minus mass of air displaced,

m' = mass of air displaced plus a possible fictitious mass,

$m + m'$ = total mass moving.

The equation of motion is

$$(10) \qquad mg - \beta_x V_x = (m + m')\frac{dV_x}{dt} = (M + \beta_2)\frac{dV_x}{dt}.$$

The droplets arrived at terminal velocities before the photographs were taken, so that $(\beta_x V_x') = mg$.

$$(11) \qquad \beta_x = mg/V_x,$$

dropping the prime.

Resistance to Motion in the Path of the Droplet.

The expression for the resistance of air, to motion in the path of the droplet is

$$(12) \qquad f(t) = f(x) \cos \theta + f(y) \sin \theta = f(x)\frac{V_x}{V_t} + f(y)\frac{V_y}{V_t},$$

where V_t is the velocity in the path of the droplet, and θ is the angle between the tangent and the X axis, and hence

$$(13) \qquad V_x/V_t = \cos \theta, \qquad V_y/V_t = \cos \theta.$$

But $f(x) = \beta_1 V_x$ and $f(y) = \beta_1 V_y$ if $\beta_x = \beta_y = \beta_1$. Therefore

$$f(t) = \beta_1 V_t. \tag{14}$$

The tangential equation of motion is

$$\left[(mg)\frac{V_x}{V_t} + (A \sin x)\frac{V_y}{V_t} \right] - \beta_1 V_t = (M + \beta_2)\left[(-a\omega^2 \sin x)\frac{V_y}{V_t} + \frac{dV_x}{dt}\frac{V_x}{V_t} \right]. \tag{15}$$

In general

$$F(t) - \beta_1 V_t = (M + \beta_2)\frac{dV_t}{dt}\Big]. \tag{16}$$

THEORETICAL AND EXPERIMENTAL VALUES OF THE ANGLE OF LAG COMPARED.

A theoretical expression for the value of γ, the angle of lag, can be deduced, based upon the above assumptions. A verification of this value by experimental results will be a strong argument in favor of these developed relations.

It has been shown that $\beta_1 = (A \sin \gamma)/a\omega$ and

$$+ \beta_2 = -\frac{\alpha}{a\omega^2} = \frac{(Ma\omega^2 + A \cos \gamma)}{a\omega^2}$$

where γ is greater than $\pi/2$. This gives

$$\sin \gamma = \frac{\beta_1 a\omega}{A}, \qquad \cos \gamma = -\left(\frac{M + \beta_2}{A}\right) a\omega^2,$$

$$tg\, \gamma = \frac{-\beta_1}{(M + \beta_2)\omega}. \tag{17}$$

But $\beta_1 = mg/V_x$ where V_x is the terminal velocity.

$$tg\, \gamma = \frac{-mg}{(M + \beta_2)V_x\omega}. \tag{18}$$

In the case of a droplet, neglect the fictitious mass β_2 and the mass of the air displaced m', so that $m = M$. Then

$$\tan \gamma = -g/V_x\omega, \tag{19}$$

a known value.

The value of V_x is found from the relation $\lambda/T = V_x$, where λ is the wave-length and T is the known period of oscillation, the frequency of the electromotive force being sixty cycles per second. The value of λ was found by measuring several successive wave-lengths with a pair of fine-pointed dividers and a steel scale.

The angle γ was found by measuring y_1 at the point on the path of the droplet where the photograph indicated the particle to be at the moment of zero field intensity. If the value of y_1 at this point be divided by the amplitude a, $\sin x_1$ is found, since $a \sin x_1 = y_1$.

The value of $(x_1 + \gamma)$ is zero or π at the point where the field intensity is zero.

$$A \sin (x_1 + \gamma) = 0 \text{ for } x + \gamma = 0 \text{ or } n\pi.$$

Since $\cos \gamma$ is negative and $\sin \gamma$ is positive, therefore π is greater than γ, which is greater than $\pi/2$.

Therefore

$$x_1 + \gamma = \pi,$$

$$\gamma = \pi - x_1.$$

Enlargements of the photographs were provided in order to make these measurements, for which a micrometer microscope was used.

In the case of the "spark gap" method of indicating the position of the droplet, the spark occurred five degrees past the position of zero field intensity on the sine curve. A correction of five degrees must be made in this manner:

$$\gamma = 180° - (x_1 - 5).$$

TABLE I.

Water Droplets.

a Cm.	λ Cm.	V_x Cm./Sec.	γ Observed.	γ Calculated.	$(M + \beta_2)/m$.
0.0837	.3410	20.50	171° 12′	172° 45′	0.825
0.2230	.1362	8.16	163° 12′	162° 18′	1.052
0.1305	.1500	9.00	165° 30′	163° 54′	1.043
0.1720	.1525	9.15	166° 30′	164° 10′	1.180
0.0977	.2830	17.00	171° 57′	171° 18′	1.088
0.0887	.1200	7.20	161° 20′	160° 10′	1.037
0.1555	.2490	14.90	168° 38′	170° 12′	0.875
0.0740	.4570	27.40	175° 42′	174° 36′	1.262
0.0337	.4360	26.10	174° 5′	174° 18′	0.965
0.0430	.3970	23.90	174° 7′	173° 48′	1.054
0.0357	.550	33.00	174° 56′	175° 30′	0.885
0.0450	.481	28.80	174° 0′	174° 53′	0.857
				Average.....	1.010
			Lycopodium.		
0.125	.346	20.7	172° 55′	172° 50′	1.02
0.075	.318	19.1	171° 20′	172° 15′	0.89

The assumptions made, that β_2 is small enough to be neglected, and that the constant $\beta_y = \beta_x = \beta_1$, are entirely justified by these results,

within the limits of experimental error. This means that if Stokes' constant ($6\pi\mu r$) holds for the vertical component velocity, it holds for the horizontal and also tangential velocities. This law will be further discussed below, after examining results for the ratio of the fictitious mass to the real mass.

RATIO OF THE FICTITIOUS MASS TO THE REAL MASS.

We have

$$\beta_2 = -\left(M + \frac{A\cos\gamma}{a\omega^2}\right),$$

from which

$$(\beta_2 + M) = -\frac{A\cos\gamma}{a\omega^2} = \left(\frac{-A\cos\gamma}{a\omega}\right)\frac{\sin\gamma}{\omega\sin\gamma} = -\frac{\beta_1}{\omega\, tg\,\gamma},$$

where

$$\beta_1 = \frac{A\sin\gamma}{a\omega}.$$

But

$$\beta_1 = mg/V_x$$

$$+(\beta_2 + M) = -\frac{mg}{V_x\omega\, tg\,\gamma}. \tag{20}$$

For the absolute value of tangent $\gamma = -c$,

$$\left(\frac{\beta_2 + M}{m}\right) = \frac{g}{V_x\omega c}. \tag{21}$$

This ratio is expressed in terms of quantities which can be obtained experimentally. According to all assumptions made, it should be close to unity for the droplets considered. A glance at the tabulated results of Table I. shows that this is true, and our knowledge of the relative sizes of M and m reveals the smallness of β_2. The ratio M/m in terms of densities, for comparative purposes, is

$$M/m = \frac{\sigma}{\sigma - \rho} = 1.002$$

approximately for water.

The wide variation from the average value which occurs for some of the droplets is because of the smallness of the amplitude a, and consequently, because of the difficulty in measuring the tangent of the angle of lag γ, accurately. The droplets with a large amplitude show more consistent results.

If one takes the average value of $(M + \beta_2)/m$ as given in the table, the ratio of β_2/m can be found for a water droplet, where $M/m = 1.002$ by equation (25).

$$\frac{M+\beta_2}{m} = 1.002 + \frac{\beta_2}{m} = 1.01,$$

$$\beta_2/m = 0.008 \text{ or } \beta_2 = .008 \text{ m}.$$

This agrees with the theoretical value calculated before.

Stokes' law is supposed to give the value of all the viscous resistance which is proportional to the velocity.[1] If such be the case, one may write $\beta_1 = 6\pi\mu r$ and make little error; since the equations deduced from the properties of the sine law show that all of the viscous resistance is given in terms of the first power of the velocity. The experimental results also verify this last conclusion. Since the resistance is proportional to the velocity, and since most of the velocities fall within the limitations of Stokes' law (as explained above) one feels justified in making the above assumption ($\beta_1 = 6\pi\mu r$) even for variable velocities.

The ratio of the charge (Ne) to the mass m can be found from these experimental results without the aid of Stokes' law; but the numerical value of β_1, the charge (Ne) and several other quantities, cannot be so determined. If this law be used to get an independent relation between the quantities involved, several unknown quantities may be evaluated in known terms. The law is stated as follows:

The viscous resistance of air for small spheres is equal to $6\pi urV$. The coefficient β_1 is then equal to $6\pi\mu r$, where μ is the coefficient of viscosity of air, and r is the radius of the sphere. The value of the mass is found from the relation,

(22) $$\beta_1 = mg/V_x = 6\pi\mu r,$$

$m = \pi r^3(\sigma - \rho)$ where σ is the density of the sphere and ρ the density of air. This gives

$$(4/3)\pi r^3(\sigma - \rho)g/V_x = 6\pi\mu r,$$

$$r^2 = \frac{6\mu V_x 3}{4(\sigma-\rho)g} = \frac{9\mu V_x}{2(\sigma-\rho)g},$$

$$r^3 = \left(\frac{(9\mu V_x)}{2g(\sigma-\rho)}\right)^{3/2},$$

$$\text{volume} = (4/3)\pi\left(\frac{9\mu V_x}{2g(\sigma-\rho)}\right)^{3/2},$$

(23) $$m = (4/3)(\sigma-\rho)\left(\frac{9\mu V_x}{2g(\sigma-\rho)}\right)^{3/2},$$

(24) $$M = (4/3)\pi\sigma\left(\frac{9\mu V_x}{2g(\sigma-\rho)}\right)^{3/2},$$

(25) $$M/m = \frac{\sigma}{\sigma-\rho}.$$

[1] G. G. Stokes, Math. and Physical Papers, Vol. III., p. 59.

VALUE OF THE FICTITIOUS MASS β_2.

For γ greater than $\pi/2$

$$-\beta_2 = M - \frac{A\cos\gamma}{a\omega^2}.$$

Transform $(A\cos\gamma)/a\omega^2$,

$$\frac{A\cos\gamma}{a\omega^2}\cdot\frac{\sin\gamma}{\sin\gamma} = \frac{A\sin\gamma}{a\omega}\cdot\frac{\cos\gamma}{\omega\sin\gamma} = \beta_1/\omega\, tg\,\gamma = mg/V_x\, tg\,\gamma.$$

Substitute

$$-\beta_2 = M - \frac{mg}{V_x\omega\, tg\,\gamma}. \tag{26}$$

Let Q equal the volume of a small sphere of radius r, $Q = (4/3)\pi r^3$,

$$-\beta_2 = Q\left(\sigma - \frac{(\sigma-\rho)g}{V_x\omega\, tg\,\gamma}\right).$$

From (24) and (25)

$$(-\beta_2) = (4/3)\pi\left(\frac{9\mu V_x}{2g(\sigma-\rho)}\right)^{3/2}\left(\sigma - \frac{(\sigma-\rho)g}{V_x\omega\tan\gamma}\right).$$

From the value of $\tan\gamma$ expressed in terms of known quantities, it is easy to calculate the ratio of the charge (Ne) upon the droplet to its weight mass m without Stokes' law. From (1), and (7*b*), the values of A and β_1 are $A = (Ne)X$, $\beta_y = \beta_1$,

$$\beta_1 = \frac{A\sin\gamma}{a\omega}.$$

Solve for A

$$A = \frac{\beta_1 a\omega}{\sin\omega}.$$

But

$$\beta_1 = mg/V_x,$$

$$(Ne)X = \frac{mga\omega}{V_x\sin\gamma},$$

$$\frac{(Ne)}{(m)} = \frac{ga\omega}{XV_x\sin\gamma}. \tag{28}$$

The value of $\sin\gamma$ in terms of $tg\,\gamma$ is

$$\sin\gamma = \frac{tg\,\gamma}{\sqrt{1+tg^2\gamma}} = \frac{g}{\sqrt{(V_x\omega)^2+g^2}},$$

$$\frac{(Ne)}{(m)} = \frac{a\omega}{XV_x}\sqrt{(V_x\omega)^2+g^2}. \tag{29}$$

The value of the charge upon the droplet can be calculated from equations (29) and (23), for from (29)

$$(Ne) = \frac{ma\omega}{XV_x}\sqrt{V_x^2\omega^2 + g^2},$$

and from (23)

$$m = (4/3)\pi(\sigma - \rho)\left(\frac{9\mu V_x}{2g(\sigma - \rho)}\right)^{3/2},$$

the value of (Ne) follows:

$$(30) \qquad (Ne) = (4/3)\frac{\pi(\sigma - \rho)a\omega}{XV_x}(g^2 + V_x^2\omega^2)^{1/2}\left(\frac{9\mu V_x}{2g(\sigma - \rho)}\right)^{3/2}.$$

Calculations of $(Ne)/m$, (Ne) and N from the data for two droplets give the following results:

a Cm.	V Cm./Sec.	X E. S. U.	Ne/m.	(Ne).	N.
.1305	9.00	21.	920	7.98×10^{-4}	1033000
.0977	17.00	7.5	1870	4.2×10^{-4}	880000

This indicates that the droplets pick up an enormous number of elementary charges under the conditions of the experiment, so that the method is valueless for the determination of e, the elementary charge.

Measurements of the Density of Lycopodium.

It was necessary to have the mean density of lycopodium for the above calculations. This was found by a porosity method. A formula for the minimum volume occupied by spheres is given in Reports of the U. S. Geological Survey.[1] The ratio of the volume occupied to the whole volume is 0.7405. The powder is composed of small seeds which are fairly regular in size, and are approximate spheres. A mass of 8.12 grams was found to indicate 19.8 c.c. when settled in a graduate. This indicated volume multiplied by the factor .74 gives 14.66 c.c. for the actual volume occupied. This gives an average density of 0.55 gms./c.c.

In John Zeleny's work on the "Fall of Small Spheres in Air,"[2] the density of lycopodium was found by a volumenometer method, which gave 1.175 gms./c.c. He found that Stokes' law was not obeyed by the powder unless this value was about twice too large. It is very interesting to find that the porosity method gives a corrected value for the density consistent with the assumption that the lycopodium powder was following Stokes' law in Zeleny's experiment.

[1] U. S. Geological Survey, Annual Reports, Vol. 19, 2, p. 301.

[2] Zeleny, Phys. Rev., May, 1910.

RESISTANCE TO THE MOTION OF SPHERES OF SMALL DENSITY FALLING IN AIR.

It was thought desirable to find by experiment, the law of air resistance to the motion of spheres falling with velocities much greater than those which are within the range of Stokes' law.

A light rubber balloon was used for the purpose. A chronograph and chronometer measured the time, and electrical means for releasing and stopping the sphere provided an accurate means for recording the time of fall on the chronograph trace. The weight in air was found to the nearest milligram. The size of the sphere was measured and the density of the air was determined from its temperature and pressure.

The data obtained in this way are given in the following table.

TABLE II.

Data for a Light Sphere Falling in Air.

$V' = 243$ cm./sec.
$m = 5.176$ gms.
$m' = 3.765$ "
Temperature 22" to 23° C.
$\beta = mg/V' = 21.7$.
$M = 8.941$ gms.
$r = 9.1$ cm.
$\delta = .00119$ gms. per c.c.

Time of Fall.	Distance.	Time of Fall.	Distance.
.0861 sec.	1.10 cm.	.469 sec.	47.0 cm.
.1084	2.3	.520	56.8
.125	3.4	.602	72.0
.138	3.6	.612	73.5
.182	6.4	.710	93.0
.243	11.2	.812	120.1
.246	12.8	.910	139.3
.317	21.7	.911	141.0
.350	24.2	1.082	173.5
.363	27.0	1.181	195.0
.381	31.4	1.389	240.5
.425	39.4	1.715	217.8
.451	42.3	1.950	373.0

From these results a space and time curve was plotted (Fig. 4). This curve is not a parabola, as it would be if the resistance did not exist. One must therefore find a new relation between time and distance fallen through, which will conform to the time-space curve found by experiment. A resistance proportional to the velocity gives the proper relation. The solid curve (Fig. 4) shows the theoretical results according to this assumption and the circles give the experimental values. The broken curve is one for no air resistance. The terminal velocity reached by the sphere was 243 cm. per sec. This is far in excess of any velocity reached by the droplets.

The mathematical development follows:

Vertical velocity at time $t = V_z$.

Terminal velocity $= V'$.

Mass of sphere minus mass of air displaced $= m$.

Mass of air displaced plus a possible fictitious mass $= m'$.

Total mass moving $= m + m' = (M + \beta_3)$.

Acceleration due to weight without air resistance $= g$.

The equation of motion is exactly the same in form as the one deduced for motion in the horizontal direction, based upon the properties of the sine-curve path of a droplet. The equation is

$$(31) \qquad mg - \beta_z V_z = (m + m')\frac{dV_z}{dt} = (M + \beta_3)\frac{dV_z}{dt}.$$

The first integral is

$$(32) \qquad V_z = \frac{mg}{\beta_z}\left(1 - e^{-\frac{\beta_z t}{m+m'}}\right) \text{ for } V_z = 0, \qquad t = 0,$$

and the second integral is

$$(33) \qquad X = \frac{mg}{\beta_z}t + mg\left(\frac{m + m'}{\beta_z^2}\right)\left(e^{-\frac{\beta_z t}{m+m'}} - 1\right).$$

It is well to note that the value of the constant β_z in this case is not given by Stokes' formula $6\pi\mu r$. Its value is $\beta_z = mg/V'$. The terminal velocity V' was 243 cm./sec., which gives

$$\beta_z = \frac{5.176 \times 980}{243} = 21.7,$$

while $6\pi\mu r = .0313$.

It is an easy matter to deduce a general relation between the variables involved when light spheres are allowed to fall in the air. It is only necessary to eliminate the exponential term in equations (32) and (33).

From (32)

$$1 - e^{-\frac{\beta_1 t}{m+m'}} = \frac{\beta_z V_z}{mg}.$$

Substitute in (33)

$$X = \frac{mg}{\beta_z}t - \frac{mg}{\beta_z^2}(m + m')\frac{\beta_z V_z}{mg}.$$

Simplify

$$X = \left(\frac{mg}{\beta_z}\right)t - \left(\frac{m + m'}{\beta_z}\right)V_z.$$

Let

$$V' = \frac{mg}{\beta_z}$$

and

$$T = \frac{m + m'}{\beta_x},$$

(34) $$X = V't - TV_x.$$

The initial conditions are $x = 0$, $t = 0$, $V_x = 0$. V' is the terminal velocity, and T is some function of the time. This is true for velocities where the resistance is proportional to the velocity. Fig. 4 shows this is true for terminal velocities as large as 243 cm. per sec.

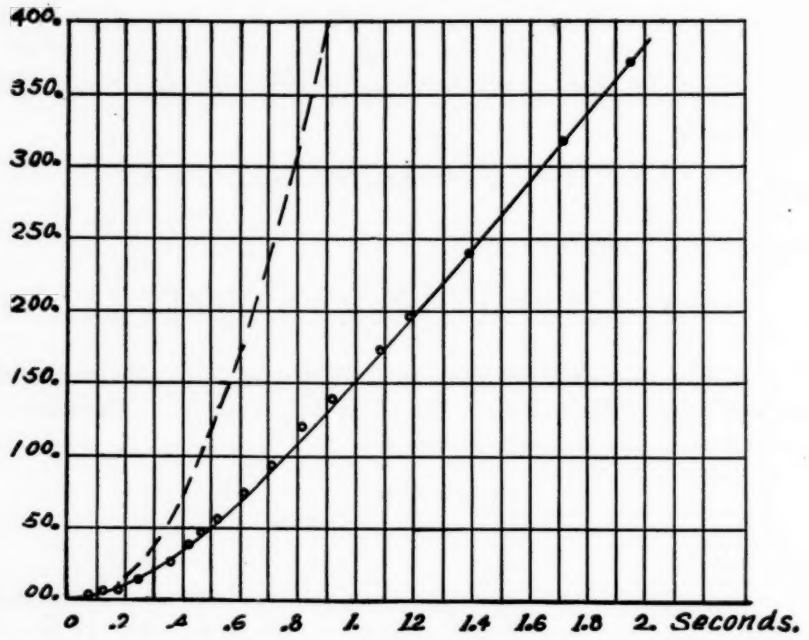

Fig. 4.

After V_x reaches terminal velocity V', TV' becomes a constant, which can be called S, representing some distance to be subtracted from $V't$, or

$$X + S = V't.$$

CONCLUSIONS.

The assumption that the vertical component of the velocity of a falling droplet oscillating horizontally in an alternating field is constant throughout the complete cycle, is verified within the limits of experimental error. The resisting force due to air is proportional to the speed in the path of the droplet.

No reason has been discovered why Stokes' constant $6\pi\mu r$ does not

apply to this case of accelerated motion, as well as to uniform motion in a straight line.

The inertia of the droplet may be increased by a fictitious mass to a slight degree, due to its motion in the air.

The droplets pick up a very large number of elementary charges, so that the experiment is useless for finding the value of e.

For much larger speeds than are within the limits of Stokes' law, the resisting force is proportional to the velocity.

In conclusion, I wish to thank Prof. E. P. Lewis under whose direction the work has been carried on, for his many suggestions.

UNIVERSITY OF CALIFORNIA,
April 15, 1918.

THE SPECIFIC HEAT OF TUNGSTEN AT INCANDESCENT TEMPERATURES.

BY PAUL FREDERICK GAEHR.

1. INTRODUCTION.

THE question of atomic heats at high temperatures has engaged the attention of various experimenters in recent years. The writer was invited last summer by A. G. Worthing to work on the question of the measurement of the specific heat, and the present paper is the result of an endeavor to work out this question independently of Worthing.

As explained by Worthing in a recent paper,[1] the specimen must be worked in a vacuum. It can not be worked in a gas-filled bulb, as then we would complicate matters. Also, tungsten is a desirable specimen to start with on account of its high melting point and slow evaporation.

There are two methods of attacking the measurement,—the direct and the indirect. All variations of them are based on the fundamental equation which states that the electrical input, when the current is charged, equals the radiation plus the heat required to raise the temperature of the filament. The result of these methods is the specific heat at constant pressure; and as we deal in this paper with only this quantity, no subscripts are used.

Corbino worked out three methods, translations of which are given in the references cited below.[2] The first two methods involve the assumption that we have available a.c. generators with a pure sine-law E.M.F. All three methods assume that the temperature changes are small enough to admit of certain expansions. In all three methods he develops equations which make possible the evaluation of certain quantities independently of a knowledge of the temperature. And while the first two methods involve many measurements of doubtful accuracy, in the third method he makes a further assumption about ballistic galvanometers,—which point is covered by Worthing in his recent paper on Atomic Heats at Incandescent Temperatures. Corbino further assumes that the "end-effects" may be neglected. This is not at all the case, however. Their evaluation is a very tedious process.[3]

[1] Worthing, PHYS. REV. Sept., 1918.

[2] Phys. Zeit., 1910, p. 413; 1911, p. 292; 1912, p. 375.

[3] Hyde, Cady, Worthing, T. I. E. S., VI., 238, 1911.

Worthing used a direct method of attack, and made a preliminary announcement several years ago.[1] He eliminated the troublesome end-effects by two methods: (1) use of two lamps identical in every detail except length of filament, and (2) application of extremely fine potential leads, of the same material as the filament, and attached at points near enough the center of the filament to get past the portions which are at a lower temperature. Also, his circuit contained a dummy lamp so arranged that while the lamp under test had its current changed, the current in the battery and most of the wiring was constant.

2. OUTLINE OF FIRST METHOD.

The "first method" consisted in changing the lamp current, and measuring, at some known time thereafter, the current and voltage of the lamp. From this were deduced the heat supply at any moment, and the temperature; and from the temperature, and the watts-temperature curve taken when the lamp was burned in the steady state, the radiation was found. Worthing simply varied his lamp between two steady states. It seemed to the writer that the time consumed in making potentiometer settings would be a drawback unless the batteries could be depended on to hold their voltage a long time. Accordingly, several changes were adopted in the hope of simplifying the measurements. These changes were:

1. The current was altered up and down by means of a motor driven-switch. On the same shaft was a contact which closed the potentiometer circuit at any desired time after a current change. We thus had a cyclic arrangement, which permitted a more rapid potentiometer setting. Also, by driving the motor fast enough, the temperature changes, and hence the radiation changes, were small; and during increasing current, the input was about three times the heat absorption of the filament.

2. Provision was made in the apparatus for the use of alternating current, should this seem desirable.

3. A small kink was put into the filament where the potential leads were attached, to prevent slipping of these leads. As the distance between the points of attachment was measured before the lamp was made up, this precaution was important.

4. It was also attempted to find the instantaneous temperatures pyrometrically.

3. THE FILAMENT AND ITS MASS.

The general appearance of the filament with potential leads is shown in Fig. 1. A number of lamps were made up, three diameters of wire

[1] PHYS. REV., N. S., V., 340, 1915.

being used for the filament, while the fine potential leads were of 1-mil wire. In the choice of diameters one makes a compromise between a diameter large enough to ensure a fairly high heat capacity per centimeter length on the one hand, and a diameter small enough to avoid considerable temperature gradients.[1]

Fig. 1.

The filaments were all carefully brought to a good polish, so that their emissivity might be pretty well known.[2] The filaments were then all weighed and measured. A point near the middle of each filament was found which was free from scratches, and the wire bent into the loop form in such a manner that this good spot should be in view at the point of the loop and from a direction normal to the plane of the loop. Finally two small V's were bent into the loop near the bottom, and the lengths from the ends to these V's measured. Subtraction of these ends from the total lengths gave the length of the part between the V's and proportionality gave the mass of that portion. In the following pages the entire loop and the part between the V's will

TABLE I.

Length and Mass of Filaments.

	Loop.		Gm./Cm.	Apex.	
	Length.	Mass.		Length.	m.
A–1	22.70	.4354	.01918	8.35	.1602
A–2	21.10	.4049	1918	7.76	.1489
A–3	21.03	.4032	1918	7.30	.1432
A–4	20.94	.4015	1917	7.96	.1526
A–5	21.85	.4191	1918	8.13	.1559
B–1	18.11	.3026	.01670	4.95	.08271
B–3	16.21	.2703	1668	5.58	.09305
B–4	15.78	.2629	1666	5.00	.08330
C–1	15.30	.06455	.004219	3.83	.01614
C–3	15.21	.06404	4210	3.95	.01580
C–4	14.96	.06300	4211	3.22	.01357

be spoken of respectively as loop and apex. The essential measurements are given in Table I. The three wires used are known respectively as 14-mil, 13-mil and 7-mil wire. It is seen from the column of linear densities that this quantity was very constant for any one wire. B-2 and C-3 were accidentally burned out.

[1] Worthing, PHYSICAL REVIEW, N. S., IV., 524, 1914.

[2] Worthing, PHYSICAL REVIEW, N. S., X., 377, 1917.

The lamps were of course all properly aged before any measurements were attempted. The lamp C-4 was used for all the experimen s on which this paper reports, as it was thought that with the smaller current less trouble would be had with the batteries.

4. MEASUREMENT OF THE TEMPERATURE.

In the place of the lamp under test (see "Lamp," Fig. 2) was put the lamp A-3, whose temperature-current curve was carefully obtained for me by Dr. Worthing at the Nela Park Laboratory. From this was calibrated the pyrometer lamp under the conditions of the later measurements. When the unknown lamp was then put in place, its temperature was easily obtainable. This substitution method has its obvious advantages.

The assumption that all filaments were polished alike, and that all the bulbs had the same absorption as the bulb of the standardized lamp, leads to possible small errors in the temperature. But these errors will be no more serious than those resulting from the uncertainty of voltage and current measurements in the first method.

In obtaining the calibration curve for the pyrometer lamp, it is well to plot the pyrometer current not only against temperature, but also against the current of the standard lamp. This second curve is much straighter than the first, and therefore is of use in finding erroneous measurements, in interpolation, and in extrapolating for a small distance. Fig. 8 shows these two curves.

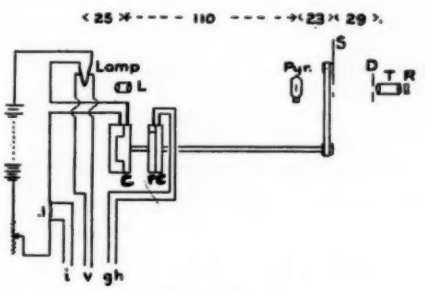

Fig. 2.

When later the unknown lamp is calibrated in the steady state, this procedure may be repeated. It is also well to plot a curve between current and voltage, as this will then show up erroneous voltage measurements, while the second curve above shows up only faulty current measurements.

Fig. 2 shows the general arrangement of the pyrometer bench. *C* and *PC* are respectively the commutator and potential contact, to be described more fully below. *L* is a 9-inch lantern objective, mounted on a carriage. The focusing of the lamp was done by means of strings within reach of the operator. *Pyr* is the pyrometer filament, situated at the image formed of the lamp by the lens *L*. The pyrometer lamp is a straight filament tungsten lamp, 10 volts, built for a galvanometer lamp.

The pyrometer filament, and the image of the lamp were viewed through the telescope *T*, which was made of sufficiently short focus by placing a spectacle lens of suitable strength in front of it. The red glass *R* is also shown, as well as the diaphragm *D*. This diaphragm is necessary to avoid diffraction.[1] The lamp could be adjusted roughly in three directions, and could also be rotated. Fine adjusting screws were provided for bringing the tip of the filament into the optic axis of the pyrometer.

A sector-disc *S* is indicated in Fig. 2. This disc was belted to an extension of the commutator shaft. To insure exact synchronism, a connecting rod was added. The coupling was then adjusted so that the lamp was visible only at the instant when the potential contact *PC* was closed, for measurement of instantaneous current or voltage of the lamp. When the commutator is run rapidly, the sector disc may be placed on the other side of the pyrometer. Of course, when the pyrometer current was adjusted for equality of brightness, this equality was approached twice from the dark side, and twice from the bright, and the average of the four currents taken. It was found even at the low frequencies used that the four settings agreed very well, although the work under these circumstances is very trying to the eyes.

The pyrometer current was passed through a standard resistance, the voltage across which was found by means of a deflection potentiometer, the details of which are given in the next section. When a pyrometer setting has been made, the operator has only to look up and read the position of a spot of light on a conveniently placed scale.

5. Measurement of Current and Voltage.

Fig. 3 shows the commutator and potential contact in greater detail. The commutator *C* was made of three rings of brass, the middle one of which was divided into segments of 60° and 300° resp. The shorter segment *bd* was attached to the right-hand ring, and the longer segment was attached to the left-hand ring. Hence when the line *a* or *e* was under the brushes, then brushes 1 and 2 were connected; but when the line *c* was under the brushes, then brushes 2 and 3 were connected. This provided for several arrangements—the current could be turned on and off, being on for one sixth or five sixths of a revolution; or the current could be varied between two limits. Or the connections could be made so as to employ a dummy. This latter was

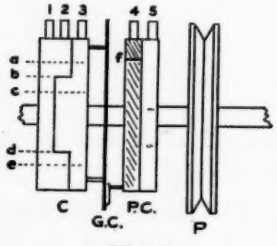

Fig. 3.

[1] Worthing and Forsythe, PHYS. REV., N. S., VI., 163, 1914.

not done with the lamp C-4, as the current was not large, and it was not thought that at the speeds employed any change of line resistance would result. The lengths of the segments on the middle ring were chosen as 60° and 300° because at first this seemed wise in view of the slow cooling of the large filament.

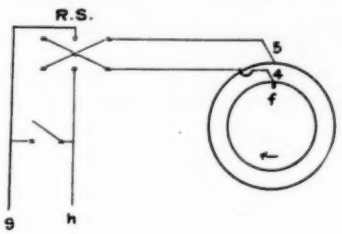

Fig. 4

The wheel *PC* consisted of a fiber disc and a brass ring. But a single narrow strip of brass *f* projected from the brass ring into the fiber as indicated. Connection between brushes 4 and 5 was thus established when *f* came under brush 4. *P* is the pulley by which the commutator and attachments were driven by the motor. To the wheel *C* was attached a graduated circle *GC*, the pointer being fastened to *PC*. Either *C* or *PC* could be turned on the shaft independently. But if *C* is turned, the motion is equivalent to moving *PC* and the sector disc together, measuring current, voltage, and temperature of the lamp at a different point of the cycle.

Quite a little trouble was experienced with parasitic E.M.F.'s associated with the potential contact *PC*. Brushes of light spring brass were finally employed, bearing very lightly on the rings. Referring to Fig. 4, it is

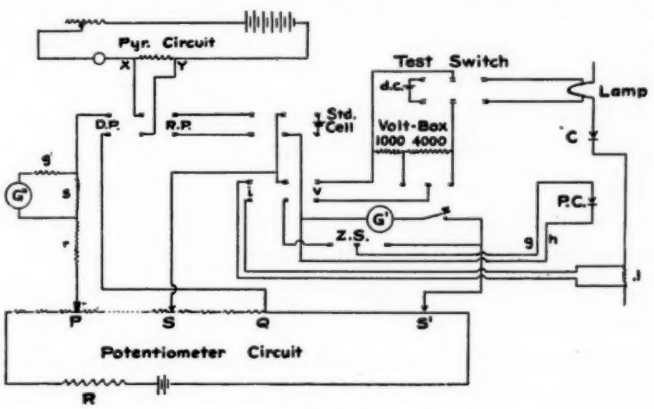

Fig. 5.

seen that a reversing switch *RS* was added, and also a short-circuiting switch. In the upper right corner of Fig. 5 is shown a test-switch, by which a dry-cell (d.c.) could be substituted for the lamp voltage. To test voltage measurements, it was merely necessary to measure the voltage of this dry-cell with *PC* short-circuited and again with the regular ar-

rangement, with the reversing switch in either position. The average of the last two readings should give the correct voltage. By this simple test the behavior of the potential contact could be tested at any time. But it was found convenient to add still another switch. In Fig. 5, just below the galvanometer G', is found the switch ZS. When ZS is closed to the left, the usual potentiometer connection is established for measurement of lamp voltage or current. But when ZS is closed to the right, we have a circuit consisting of the galvanometer and the potential contact. The parasitic E.M.F. would then move the galvanometer to some position which can be taken as the zero-point; ZS is then quickly thrown over, and if the potentiometer is correctly set, no change in deflection results. For ZS was used a relay, with an insulated extra contact substituted for the "back-stop." The relay was operated by a key worked by one's knee, leaving the hands free for the potentiometer and the usual galvanometer key. This relay switch possesses the advantage of changing ZS from one position to the other quickly and without introducing any further E.M.F. from friction of the knife-blade in the jaws, or from the heat of the hand. Finally, a 2-mf. condenser was connected across the switch-posts v (Fig. 5), nearly trebling the accuracy of adjustment. Careful tests, extending over runs of much over an hour showed the accuracy of voltage and current measurements to be about 1/10 per cent.

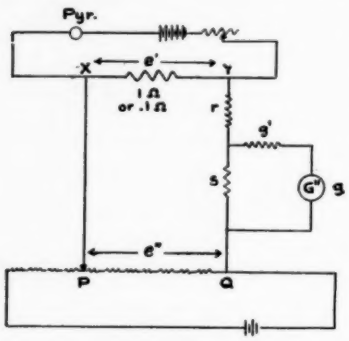

Fig. 6.

The length of time during which contact is made on PC is about 1.5 per cent. of the time necessary for the segment bd on the commutator to pass under the middle brush. The hole in the sector-disc S transmitted light for just about the same length of time. The duration of the contact PC is about .0008 second with the commutator running at 200 R.P.M.

There was also a switch for short-circuiting the commutator, by which it was possible to turn the sector-disc without altering the lamp current. Several careful tests showed the same potentiometer setting with the disc running and standing still, but the settings in the former case were harder to make.

6. The Potentiometer.

Several portions of the potentiometer table have already been described in detail. In the middle of Fig. 5 we find two $DPDT$ switches for connecting the standard cell, the voltage leads from the lamp, the voltage

leads from a 0.1 ohm resistance in series with the lamp, or the voltage leads from a standard resistance in the pyrometer circuit. The resistance of the coils in the volt-box are given, and it is seen that the lamp voltage could be measured with or without the use of this box. The commutator and potential contact are indicated at the right. It is shown in the figure that when lamp current or voltage are being measured, the potential contact is in series with the galvanometer.

The *DPDT* switch in the upper left corner of Fig. 5 must be closed in the *RP* position (regular potentiometer) when the pyrometer current is to be measured in the regular way. When this switch is closed in the *DP* position (deflection potentiometer), the pyrometer current is measured by the deflection potentiometer. This switch must not be left closed in the *DP* position when any of the other E.M.F.'s are connected to the potentiometer.

The potentiometer used was the Leeds and Northrup "Student Potentiometer," which was considered sufficiently accurate, and has a larger range than the others. The galvanometer G' is one of the Leeds and Northrup High Sensitivity Type; it was really too sensitive for the potentiometer except when the instantaneous currents and voltages were measured.

A binding post Q was added, and a travelling plug P was made. A galvanometer G'' was set up to reflect the light to same scale as used for G'; G'' was surrounded by resistances r, s and g', as shown more clearly in Fig. 6. If e' and e'' be the p.d.'s across XY and PQ respectively, the resistances r, s, g' and the scale, can be arranged to give the difference $e'-e''$ directly in millivolts. s and g' were chosen to make the galvanometer G'' practically dead-beat, and r was then adjusted to make the scale reading equal to $e'-e''$ in milli-volts. In practice the approximate value of e' is found from an ammeter in the pyrometer circuit, and the plug is placed in the tenth-volt stud reading nearest the probable value of e'.

Let the resistance PQ be a ohms, and the rest of the potentiometer circuit be b ohms. Then write $a' = ab/(a + b)$. The similar function in the pyrometer circuit may be called 1 ohm, since that was its approximate value in our case. Also let $g'' = s(g + g')/(s + g + g')$. Then e' and e'' being the p.d.'s across XY and PQ respectively when the connection between the two circuits is broken, the current flowing through the resistance r, when the connection is reëstablished is

$$I_g = \frac{e' - e''}{a' + 1 + r + g''}.$$

From this equation it is evident that if the denominator be kept constant for all positions of the travelling plug P, and for all values of the rheostat in the potentiometer circuit, then the galvanometer current and deflection are proportional to the difference $e'-e''$. If a sensitive galvanometer is used, then r will be so high that the current passing from one circuit to the other will always be very small, and the connection between the two circuits may be left on while the pyrometer settings are being made. In other words, the presence of the current I_g will not appreciably alter the pyrometer current. In our case, the maximum value of I_g was only .000005 amp.

In the foregoing equation it is necessary to keep only $a' + r$ constant, as the other values are not changed.

A change of the potentiometer rheostat changes the value of a' for any a. So as the battery voltage changes, and the rheostat has to be changed, a new table of r's must be prepared. But calculation will show how much the rheostat may change before any appreciable difference in the galvanometer deflection results. In our set of resistances, in the worst case the rheostat could be changed by as much as 20 ohms before a new table of r's had to be prepared. Such tables were then computed for the entire range of the battery rheostat corresponding to the possible changes in battery voltage. If the galvanometer changes its constant for any reason, the proper proportionality is again established by changing the value of g'. This will not affect the value of g'' appreciably.

The galvanometer G'' was also one of Leeds and Northrup make, not quite as sensitive as the other, of higher resistance, free from zero-drift, and possessing good proportionality of deflection to current. Two galvanometers were used for our potentiometer because each was especially adapted to its work. G'' was not sensitive enough to measure the instantaneous values, and G' did not deflect in proportion to its current.

Using a 1-ohm standard up to 2.25 amps in the pyrometer circuit, we found that 1 milli-amp change in the current (*i. e.*, 1 milli-volt change in potentiometer reading) corresponded to about 1.2 degrees change in lamp temperature. As our deflection potentiometer could be easily read to 1/2 scale division, the uncertainty of temperature from this source was about .6 degree. For the higher currents a 0.1 ohm standard was used, but in this case the deflections had to be estimated to 1/10 scale division.

In order to avoid having two resistance standards of the usual type on the table, and to escape the necessity of changing connections, a special resistance standard was constructed. Fig. 7 shows its construction. At the left are shown two pairs of wires, representing respectively current

and potential leads. The current leads are of No. 10 copper wire. When the switch at the right is in the upper position, the current traverses only the .5 ohm coils. These are of no. 20 Therlo wire, and are adjusted until the voltage between the potential leads is just a shade higher than the drop across a standard tenth-ohm in series with this device. The final

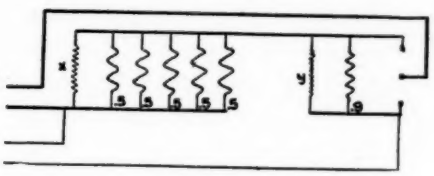

Fig. 7.

correction is best made by means of the german silver wire x. Similarly, when the switch is in the lower position, the current traverses the entire apparatus, and the .9-ohm coil (also of no. 20 Therlo wire) is adjusted in comparison with a standard ohm—the final correction being made by the coil y. The therlo wires are arranged in short vertical coils, and are therefore air-cooled. Careful tests showed that the .1-ohm arrangement

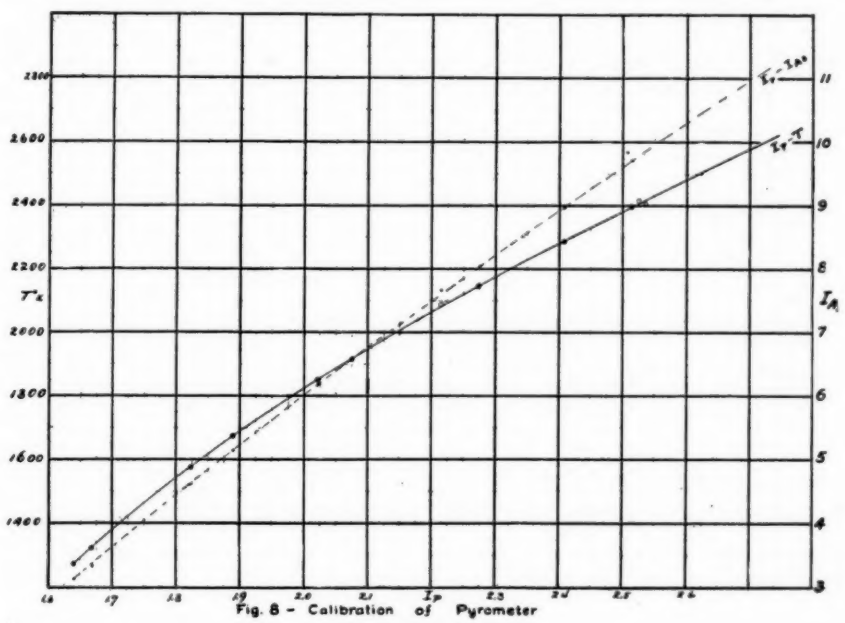

Fig. 8.

was good up to 12 amps, and the 1-ohm up to 5 amps, the error being less than 1/5 per cent. This changeable standard was constructed by the writer for use, in connection with the deflection potentiometer, in the rapid checking of ammeters.

7. THE MOTOR. SPEED REGULATION.

The motor was built as a converter, and was chosen because it could be used as a generator of alternating current. Its speed could be kept nearly constant by connecting an inductive load, or a bank of tungsten lamps, to the a.c. armature. A portion of the shunt-field resistance was within reach of the operator sitting at the telescope of the pyrometer. The series field was connected compound. After running about 10 minutes, it kept its speed pretty well, but it had to be watched.

In order to know at each instant the speed of the motor, a Stewart-Warner speedometer was attached, the gearing employed being such that with a motor speed of 1,090 R.P.M. the speedometer indicated 20 mi./hr. Careful tests revealed that strict proportionality between speedometer reading and motor speed does not exist, and that the reading appeared to be somewhat dependent on the temperature. The speedometer was finally used only to indicate the constancy of speed, and the actual commutator speed corresponding to a speedometer reading was obtained each time that a run was made.

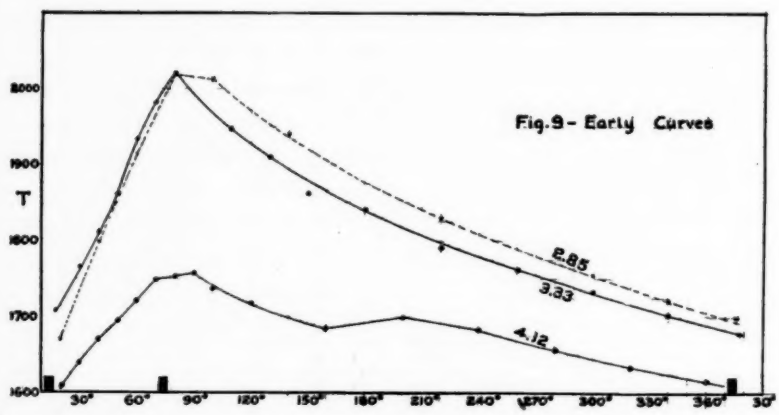

Fig. 9.

It was hoped to build the apparatus so that the potential contact and the sector-disc could be shifted to different parts of the cycle without stopping the motor, but this did not seem advisable due to lack of time. While the motor was standing still, the resistance of lead wires and other copper would cool down, and after starting up again, the motor had a rather different speed. Likewise, during the off period, changes occurred in the lamp circuit. So that it became necessary to wait several minutes after each change before readings could be resumed.

Fig. 9 shows some trial curves taken early in the experiment. They

show the rise and fall of the lamp temperature during a cycle, the temperatures being obtained by the pyrometer.

TABLE II.

First Method—Data for Fig. 11.

C-4, 3.18 cycles/sec.

ϕ.	I.	V.	Pyrometer.	T by Pyrometer.	T by Resistance.	k.
16°	1.000	1.117	1.9738	1790	2575?	
26°	3.08	2.280	1.9804	1799	1818	0369
36°	3.11	2.3325	1.9953	1819	1836	0376
46°	3.093	2.35	2.0133	1840	1857	0408
56°	3.090	2.3775	2.0298	1861	1873	0428
66°	3.078	2.386	2.0403	1881	1888	0478
72°	3.065	2.400	2.0513	1889	1902	
77°	1.608	1.55	2.0503	1888	2068?	
90°	1.605	1.255	2.0433	1878	1899	0401
135°	1.605	1.2425	2.0303	1862	1886	0394
180°	1.615	1.23	2.0235	1853	1860	0395
225°	1.610	1.219	2.0098	1836	1850	0447
270°	1.610	1.206	1.9840	1803	1832	0451
315°	1.615	1.206	1.9825	1800	1830	0386
360°	1.647	1.213	1.9753	1792	1812	0324
12°	1.630	1.203	1.9720	1788	1815	

During these runs, no special attempt was made to keep the motor speed constant. In fact, the speed changed by about 4 per cent. It is hoped in the near future to alter the construction of the apparatus, and try this method again.

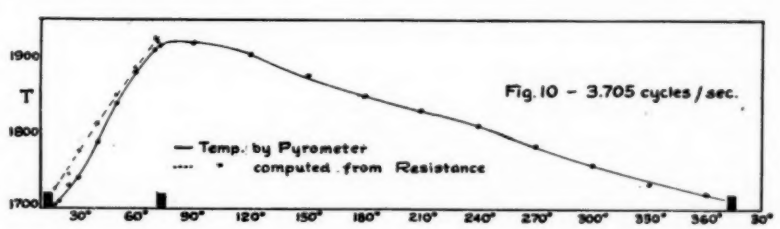

FIG. 10.

Figs. 10 and 11 show much better runs. There appears to be a bad variation in the temperature from the theoretical curve, but it should be noticed that the temperature scale in Fig. 11 is much more open than in the other curves.

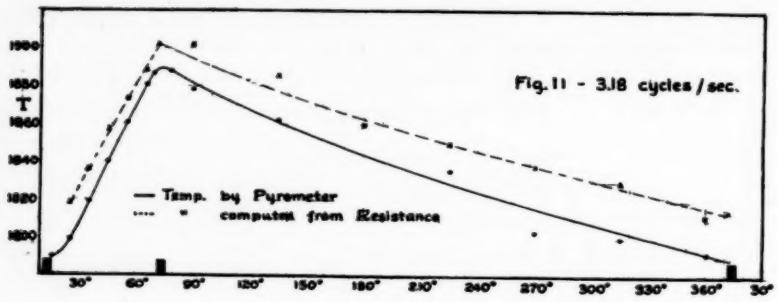

Fig. 11.

8. Temperature Variation of the Lamp During One Cycle.

The number of cycles per second in the runs shown in Figs. 9–11 are indicated in the figures. In Figs. 9 and 10 we represent runs in which the current was entirely off during five sixth of the cycle. It was noted in several curves that the rising portion showed a decided inflection. As this is contrary to the theory, it was decided to compute the temperature from the instantaneous resistance, as other observers have done. The dotted curves in Figs. 10 and 11 show these computed temperatures, and in the dotted curves we find no inflection. In order to be able to compute the temperature during the entire cycle in this way, it was necessary to have current during the entire cycle, and so Fig. 11 shows a run in which the resistance of the rheostat was varied between two values. The three heavy marks on the base line show the points at which the middle brush crossed from the short to the long segment, and vice versa.

The discrepancy between pyrometric and resistance methods of determining the temperatures is probably due to the fact that the filament is in rapid motion, thus decreasing the brilliancy of the image by spreading it out. In other words, the hole in the sector-disc was not "instantaneous." The writer will take up this particular question again at a later date. It is observed that the difference between the dotted and full curves in Figs. 10 and 11 is not constant. It can hardly be said that the lower end of the filament is stationary at the transition points of the current from one part of the cycle to the other. For at the speeds employed, there are undoubtedly heat exchanges between the filament and the leading-in wires, which lag behind the current changes. If one views the filament while the motor speed is slowly changing up and down, the filament is seen slowly to move up and down. As higher motor speed decreases the temperature range, hence at any point of the cycle the distance of the filament from its mean position will be less than when the motor is running more slowly.

It must be clear that a change in the motor speed alters also the current consumption, hence the resistance of the rheostats and leads from the battery. Hence it is clear that if a cyclic method is adopted, it is absolutely essential to make sure that conditions be kept constant.

9. The Calculations.

In view of the difficulties encountered it was decided to compute the temperature from the instantaneous resistance, and to draw smooth

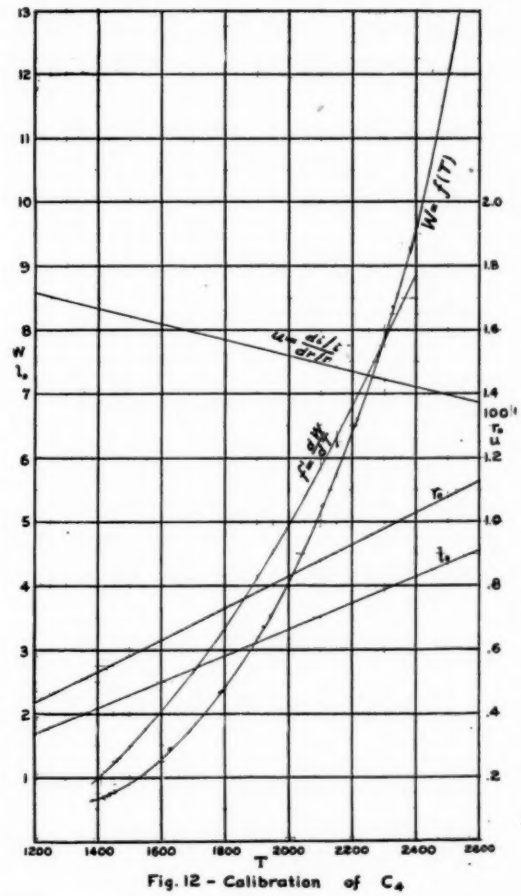

Fig. 12.

curves through the points. It must be remembered that in this first method it is necessary to know the rate of change of temperature, that therefore we must have a curve representing apparently a single cycle.

From data in Tables III. and IV. the curves in Fig. 12 were constructed. At present we are interested only in the watts and the apex resistance.

The watts being obtained with the lamp burning in the steady state, are therefore equal to the radiation. If W and w are the electrical input and the radiation resp., and c is the heat capacity of the filament, then

$$c = \frac{W - w}{d\theta/dt}.$$

We therefore find $d\theta/dt$ from a curve like the dotted curve in Fig. 11, and for the same temperature the w is found from Fig. 12. W is of course measured directly. In this way the value of c is found at various points along the temperature curve. Then c is divided by the mass of the filament, and by 4.19, the result being the specific heat in calories per gram.

The above equation was changed slightly in order to be more convenient. Let φ be the angular position of the commutator, and s its speed in revolutions per minute. Then

$$d\varphi/dt = 6s/10,$$

the unit of angle being 10°. Now

$$d\theta/dt = (d\theta/d\varphi) \times (d\varphi/dt),$$

and $c = km$. Hence we have for a working equation

$$k = \frac{W - w}{d\theta/d\varphi \times d\varphi/dt \times m \times 4.19} = \frac{W - w}{d\theta/d\varphi} \times \text{const.}$$

The temperature curves are plotted against φ instead of against time, and the slope is found as the rise per 10° of the tangent at any point.

TABLE III.

Calibration of Lamp C-4. (Fig. 12)

I.	*V.*	Pyrometer.	*T.*	*W.*	*R.*
1.098	.5875	1.7002	1383	.6451	.5351
1.154	.637	1.7303	1440	.7351	.552
1.575	.95	1.8546	1628	1.495	.635
1.7999	1.3055	1.976	1791	2.3498	.72531
2.020	1.581	2.0673	1906	3.1936	.78267
2.060	1.6320	2.0757	1919	3.3619	.7922
2.095	1.6770	2.0953	1942	3.5133	.8005
2.653	2.484	2.337	2215	6.59	.93628
2.924	2.9285	2.463	2342	8.5628	1.0015
3.013	3.082	2.496	2383	9.286	1.0229
3.474	3.830	2.7002	2544	13.305	1.1025

The first two and the last two columns refer to C-4, its temperature being determined from the pyrometer reading.

The results obtained from the data in Table II. are given in Fig. 13. There is no definite reason to believe from these data that there is any difference in k between the heating and cooling values. The small circle in Fig. 13, designated as the average of all points by the first method, was plotted with coördinates equal respectively to the average k and average T of all the points by the first method. It is seen that this average point lies very close to the straight line drawn through the points obtained by the second method.

Attention is called to the fact that in the case of the data in Table II., abnormally high resistance temperatures were obtained at the two points just after the middle brush on C had slipped from one segment to the next. No explanation is offered for this. The writer had worked out the details of the second method, and this looked so promising that no further data were taken with the first method.

Before going to the next method, it may be well to say that the writer has obtained in calibrating C–4 also the voltage across the entire lamp. Plotting the data in Tables III. and IV., straight lines were found for both r and l, these being the resistances of the apex and the entire loop respectively. We shall in later discussions have occasion to speak of temperature variations defined by $T = T_o + \theta$, and of resistance variations defined by $r = r_o\,(1 + a\theta)$ and $l = l_o\,(1 + a'\theta)$, in which of course r_o and l_o are the resistances at T_o. It was found that $r = .0004973\ T - .16276$ and $l = .00285\ T - 1.149$. From these relations can be found the T_o corresponding to any r_o. Also, it is easily shown that $1/a = T_o - 325$, and $1/a' = T_o - 525$. These equations will be of use later.

10. Theoretical Considerations.

In addition to the symbols already employed, we will designate the E.M.F. of the battery by E, and let the resistance of the circuit, exclusive of the lamp, be respectively R and $R - R'$ in the two parts of the cycle. The mean temperature of the lamp during the cycle is taken as T_o, and its resistance at that time as r_o. During the first part of the cycle, when the current is increasing, we may write the equation of heat exchanges thus

$$W_1 = \frac{E^2 r_o(1 + a\theta)}{[R - R' + l_o(1 + a'\theta)]^2} = W_o(1 + a\theta)\left[1 - \frac{R' - a'\theta}{R + l_o}\right]^{-2} = f(T_o) + \theta.f'(T_o) + c\frac{d\theta}{dt} \qquad (1)$$

W_o is *not* the input at the mean temperature. $f(T)$ has already been expanded to a two term expression involving only first powers of θ. If we do the same with the $[\ ]^{-2}$ term, we obtain

$$c\frac{d\theta}{dt} = \left[2W_o\frac{R'}{R+l_o} + W_o - f_o\right] - \theta\left[f_o' - W_o a\left(l - \frac{2l_o}{R+l_o}\cdot\frac{a'}{a}\right)\right] \quad (2)$$
$$= A - B\theta.$$

Putting β for B/c, and taking τ as the duration of the first part of the cycle, and the initial and final temperatures as $T_o \pm \theta_o$, we obtain as solutions

$$\beta\tau = \log\frac{A + B\theta_o}{A - B\theta_o} \quad \text{or} \quad \theta_o = \frac{A - (A + B\theta_o)\epsilon^{-\beta\tau}}{B}. \quad (3)$$

In the second part of the cycle, the rheostat and line resistance are R, and the duration of the second part is 5τ. The solutions are similar except for the fact that the term $2W_oR'/(R + l_o)$ is absent, and the new constant A' differs from A by that amount. The corresponding solutions are therefore

$$5\beta\tau = \log\frac{A'}{A' - B\theta_o} \quad \text{or} \quad \theta_o = \frac{A'}{B}(1 - \epsilon^{5-\beta\tau}). \quad (4)$$

From (3) and (4) we then obtain the two equations

$$\left(\frac{A + B\theta_o}{A - B\theta_o}\right)^5 = \frac{A'}{A' - B\theta_o}, \quad (5)$$

$$1 + \frac{2W_oR'/(R+l_o)}{W_o - f(T_o)} = \frac{1+\epsilon^{\beta\tau}}{1-\epsilon^{\beta\tau}}(1 - \epsilon^{-5\beta\tau}) = -\coth\frac{\beta T}{2} \times (1 - \epsilon^{-5\beta\tau}) \quad (6)$$

If the exponential term $\epsilon^{-5\beta\tau}$ can be neglected with respect to unity, equation (6) can be used to find β, hence c and k. It is doubtful whether this method of finding k is practical as it seems easier to use the direct measurements. Also, this way of finding k requires temperature fluctuations so small that the several expansions hold good (*i. e.*, θ_o should not exceed 10°); it is also necessary that the rheostats should have a zero temperature coefficient of resistance, and that the battery should be absolutely constant. This method also would require that the transition from one part to the next should not be made by a wide brush slipping from one segment to the next, but by a snap switch. However, the equations have a theoretical interest in that they point out that the temperature rise and fall in such cases is exponential, *i. e.*, that there is no overshooting. The equations also show that the variation of electrical input during one part of the cycle is necessarily small.

As to the expansion of $f(T)$ to only two terms, we observed from a study of the watts curve that the relative errors committed in the substitution of $f'(T_o)$ for $[f(T) - f(T_o)]/\theta$ are 0, 3.1 and 13.2 per cent. for $\theta = 20°$, 50°, 100° resp. in the case of the lamp C-4. In the computation of B,

the error resulting from this assumption is practically the error in $f'(T_o)$.

If in equation (1) we expand $f(T)$ and the $[\ ]^{-2}$ term to second powers in θ, we will get an expression, the analysis of which shows that there may be an inflection in the temperature curve when the temperature range is large.

If the source of current is an alternating E.M.F., the case is somewhat different. In the integration of the equation corresponding to (1), we can not neglect the "initial term," which under certain circumstances may be large enough to permit an overshooting of the temperature.

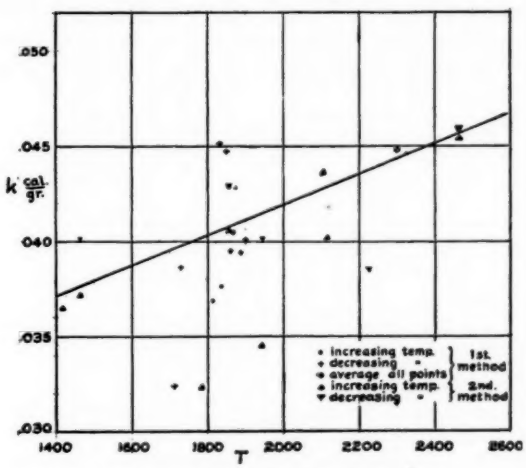

Fig. 13 – Specific Heat of Tungsten

Fig. 13.

Equations (3) and (4) show how the temperature variations may be predicted under conditions similar to those of this experiment.

Should it ever be desirable to measure the specific heat of some substance whose temperature coefficient of resistance is very small, it will probably be necessary to employ the pyrometric method of getting instantaneous temperatures. It is believed that this will be possible if proper precautions be taken to get "instantaneous" flashes. Probably a photometric method would be better.

II. The Second Method.

The second method is really an adaptation of Corbino's third method to the type of lamp used, with further simplifications. Let us first suppose that the potential leads from the lamp are connected to the potentiometer, and that this has been balanced. Now short-circuit a small portion of the resistance in the lamp circuit, thus changing the current

and the resistance of the lamp. In the case of tungsten, this change is an upward one. Balance the potentiometer by short-circuiting a portion of its rheostat (see Fig. 14). The two switches S and S' can now be opened again—the galvanometer key being left closed. Although after a little the potentiometer will be in balance, as we have only returned to the former condition, yet during the time that the lamp is dropping to the lower temperature the potentiometer will be unbalanced, and we obtain a throw which can be used to find the specific heat. It is not

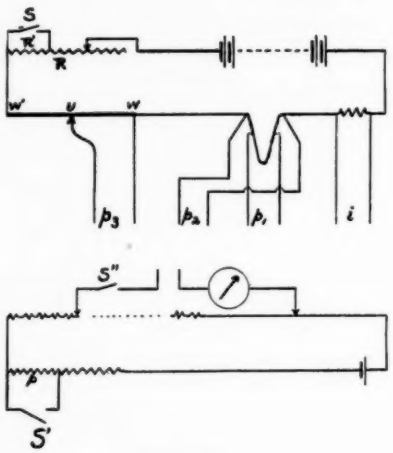

Fig. 14.

Showing arrangement of potentiometer for the second method. i are the leads for measurement of current. p_1, p_2, p_3, are the leads for measurement of apex and loop voltage, and of I_1r_0. The volt-box is omitted from the diagram.

necessary to use the switch S', in fact it is more convenient not to do so. The potentiometer can be set with S closed, the galvanometer key opened, S opened, S closed again after about 10 seconds, the galvanometer key closed within a known time later—as will be presently explained.

We will use symbols in their former meaning, except that the subscript zero now refers to the lamp at the lower steady state, and that the letter S will be used to designate the slope of a certain straight line. The subscripts 1 and 2 refer to the heating and cooling periods of the lamp.

When burning in the lower steady state, the lamp is consuming and radiating energy according to the equation

$$W_o = \frac{E^2R}{(R + l_o)^2} = f(T_o).$$

(It will not be necessary to retain the subscripts in the case of r and l.) Now close S and S', taking the time of closing as the origin of time. The input during the change of the filament resistance to its new value is .

$$W_1 = \frac{E^2 r(l + a\theta)}{[R - R' + l(1 + a'\theta)]^2} = f(T_o) + \theta \cdot f'(T_o) + c\frac{d\theta}{dt}.$$

Expanding as in equation (1), but having $W_o = f(T_o)$ this time,

$$c\frac{d\theta}{dt} = \frac{2W_o R'}{R + l} - \left[f'(T_o) - W_o a\left(1 - \frac{2l}{R + l}\cdot\frac{a'}{a}\right)\right]\theta = A - B\theta. \quad (7)$$

Similarly on opening S and S', we have during the second change

$$W_2 = \frac{E^2 r(1 + a\theta)}{[R + l\,(1 + a'\theta)]^2}, \quad \text{and} \quad c\frac{d\theta}{dt} = -B\theta. \quad (8)$$

The solutions of (7) and (8) are easily obtained. At times 0 and infinity we know that $\theta = 0$ and θ_o respectively, in the first of the two equations; and just the reverse in the other. Hence we have.

$$\theta_1 = \frac{A}{B}(1 - \epsilon^{-\beta\tau}) \quad \text{and} \quad A/B = \theta_o. \quad (9)$$

$$\theta_2 = \theta_o \epsilon^{-\beta\tau}. \quad (10)$$

If the voltages across the filament apex during the first and second changes be designated by V_1 and V_2 respectively, we have

$$V_1 = \frac{Er(1 + a\theta)}{R - R' + l\,(1 + a'\theta)} = \frac{Er}{R + l}\left[1 - \frac{R' - a'\theta}{R + l}\right]^{-1}(1 + a\theta)$$
$$= V_o\left[1 + a\left(1 - \frac{l}{R + l}\cdot\frac{a'}{a}\right)\theta + \frac{R'}{R + l}\right], \quad (11)$$

$$V_2 = V_o\left[1 + a\left(1 - \frac{l}{R + l}\cdot\frac{a'}{a}\right)\theta\right]. \quad (12)$$

During these two changes, the potentiometer had been set for the final values of V_1 and V_2 respectively, which values we will designate by e_1 and e_2. Let the "multiple resistance" of r and the rest of the lamp circuit be r', and the similar function in the potentiometer circuit be r'', the galvanometer resistance being g, then the galvanometer current in the two cases is

$$G = \frac{e_1 - V_1}{r' + r'' + g} \text{ or } \frac{e_2 - V_2}{r' + r'' + g} = \frac{V_o a\left(1 - \frac{l}{R + l}\cdot\frac{a'}{a}\right)\theta_o}{r' + r'' + g}\cdot\epsilon^{-\beta\tau} = G_o\epsilon^{-\beta\tau}, \quad (13)$$

and the total charge passing through the galvanometer while r acquires its new resistance is found by integration:

$$Q = \int_0^\infty G\,dt = \frac{G_o}{\beta} = Q_o\delta. \quad (14)$$

The ballistic constant Q_o depends on two things—the damping, and the

length of time that any appreciable current flows through the galvanometer. This constant Q_o can be found.[1] Corbino thought that by taking a ballistic galvanometer with a period of over 30 secs. the Q_o a found by condenser discharge (with correction for damping) could be used. This is certainly not the case with filaments like those which Worthing and I have used. $\beta = B/c$ can be found after G_o is determined, and from β we can get c after finding B. And finally, $c = km$.

To find G_o, we must have a current equal to the initial value of G, (13). Inspection of (11), (12), (13) shows that we will obtain a steady current equal to G_o if S is closed and the potentiometer is set for the value

$$e' = V_o\left[1 + \frac{R'}{R+l}\right].$$

In this case, the galvanometer current is equal to

$$\frac{V_o\left[1 + \frac{R'}{R+l} + a\left(1 - \frac{l}{R+l}\cdot\frac{a'}{a}\right)\theta_3\right] - V\left[1 + \frac{R'}{R+l}\right]}{r' + r'' + g} = G_o. \quad (15)$$

If we had a lamp for which $a = 0$, then the final value of V_1 would be equal to e'. While we can find e' by multiplying the final value of I_1 by r_o, it is easier to include in the lamp circuit a slide-wire of some wire like Therlo, capable of carrying the currents used, and adjusting the slider so that the resistance included between the potential leads p_3 is equal to r_o. The voltage across this portion of the slide-wire, when S is closed, is then equal to e'. Fig. 14 indicates the potentiometer arrangements covering the second method.

To find B, we may proceed directly to measure the various quantities involved. Corbino suggests a method of finding B in the case of lamps whose temperature is not definitely known.

$$f'(T_o) = dW_o/dT_o = dr_o/dT_o \times dW_o/dr_o = ar_o dW_o/dr_o.$$

Also since $W_o = i_o^2 r_o$, we find that

$$ar_o\frac{dW_o}{dr_o} = aW_o\left[1 + 2\frac{di/i_o}{dr/r_o}\right].$$

Hence

$$B = f'(T_o) - W_o\left[a - \frac{2a'l}{R+l}\right] = 2W_o a\left[\frac{di/i}{dr/r} + \frac{l}{R+l}\cdot\frac{a'}{a}\right]. \quad (16)$$

The first form for B is the better one to use—as it is not necessary to observe any precautions about small values for di.

[1] Worthing, PHYS. REV., N. S., VI., 165, 1915.

12. Direct Determination of β.

The value of β, and hence of c and k, can be found more easily than by the application of the equation (14). And it is this more direct evaluation of β which forms the chief divergence of the "second method" from Corbino's third method.

Returning to (13), we may write

$$G_1 = G_o \times \epsilon^{-\beta\tau_1} \times e^{-\beta(\tau-\tau_1)}. \qquad (17)$$

If the galvanometer key is closed t_1 seconds after S has been opened or closed, the current which will then flow through the galvanometer is clearly represented by (17), and the charge which will flow through the galvanometer in the remaining time is

$$Q_1 = \int_{\tau_1}^{\infty} G_1 dt = \frac{G_o}{\beta}\epsilon^{-\beta\tau_1} = Q_o \cdot \epsilon^{-\beta\tau_1} = Q_o \cdot \delta_1. \qquad (18)$$

Table IV.

	1.	2.	3.	4.	s_1.	t_1.	
1 I_o	1.125	1.798	2.436	2.901	19.6	.495	
2 $I_o + \Delta I_o$	1.159	1.830	2.445	2.913	13.8	.838	
3 V_r	.6090	1.3042	2.1526	2.8855	13.8	.892	1
4 $V_r + \Delta V_r$	.6391	1.3452	2.1633	2.9055	7.8	1.41	
5 V_l	2.344	5.2975	8.930	12.1	6.0	1.58	
6 E	31.2	32.0	31.0	30.9			
					42.3	0	
7 l_o	2.0836	2.9463	3.666	4.000	11.3	.535	
8 r_o	.5413	.7254	.88365	.99745	10.0	.594	2
9 $r_o + \Delta r_o$	.5513	.73502	.88476	.99467	8.2	.614	
10 T_o	.1416	.1786	.2104	.2327	4.8	.813	
11 a'/a	.82	.87	.89	.90	3.2	.936	
12 $l/(R + l)$	.075	.171	.289	.393			
13 11 × 12	.059	.149	.264	.353	6.0	0	3
14′ u'		1.34	1.94	2.47)	.4	.639	
14 u	1.62	1.58	1.496	1.437	1.0	0	4
15 13 + 14	1.679	1.730	1.760	1.780	.1	.391	
16 W_o	.6851	2.345	5.244	8.371			
17 B	.00211	.00555	.01036	.0149			
18 s	.441	1.31	1.82?	2.56?			
19 k	.0365	.0323	.044?	.045?			

From r_o were obtained T_o, a, and a' as explained in 9.

u' was computed according to the second form of B directly from the data. (eq. (16).)

u was obtained by calculation according to (21).

Third and fourth values of s and k are in doubt. Results are not plotted in Fig. 13.

All throws obtained on increasing current.

$$k = \frac{.4343}{4.19m} \times \frac{B}{s} = 7.65 \times \frac{B}{s}. \quad (20)$$

TABLE IV *Continued.*

	1.	2.	3.	
1 I_o	1.1955	2.1022	2.8321	For this set and the next, a precision potentiometer was used, and S was opened and closed. The two values of δ_1, s and k corresponding to rising and falling current are given in their proper places, the values for rising current being given first.
2 $I_o + \triangle I_o$	1.2025	2.1225	2.8658	
3 V_r	.6745	1.6896	2.7759	
4 $V_r + \triangle V_r$	.68095	1.7164	2.8349	
5 V_l	2.624	6.949	11.7	
6 E	32.2	32.1	31.1	
8 r_o	.56420	.80375	.98015	
9 $r_o + \triangle r_o$	.56626	.80870	.98920	
10 T_o	.1462	.1943	.2298	
13 $\frac{l}{R+l} \cdot \frac{a'}{a}$	.0665	.119	.343	$l/(R+l)\,.\,a'/a$ was taken from a curve. u' is here shown as in the preceding data to illustrate the difference in the two methods of calculating u. With the precision potentiometer the agreement is much better, but great care is necessary.
(14′ u'	1.60	1.56	1.29)	
14 u	1.66	1.525	1.445	
15 (13) + (14)	1.722	1.734	1.788	
16 W_o	.8055	3.55	7.86	
17 B	.00244	.00757	.0142	
18 s	.504 .466	1.68 1.44	2.42	
19 k	.0372 .0401	.0345 .0402	.0448	
t_1	S_1			
.196	4.19 4.9	4.6	2.9	
.277	3.95 —	3.1	1.2	
.370	3.52 4.03	2.05 2.05	.8	
.477	3.12 —	1.2 1.6		
.562	2.95 3.38			
.678	2.63 —			
.777	2.1 2.9			

Taking logarithms of both sides, we then find the working equation

$$\log \delta_1 = -\beta t_1 + \text{const.} \qquad (19)$$

from which it is clear that if we obtain the throws by closing the galvanometer key at different times t_1 after the manipulation of S, and plot $\log \delta_1$ against t_1, we shall have a straight line. As it is easier to look up the common logarithm, we will have an equation similar to (19) except that βt_1 is multiplied by .4343.

TABLE IV.—*Continued.*

	1.	2.	3.	4.
1 I_o	1.9197	2.4518	2.6784	3.1761
2 $I_o + I$	1.9513	2.4874	2.7215	3.2253
3 V_r	1.4569	2.1794	2.5279	3.3767
4 $V_r + V_r$	1.4965	2.2331	2.5969	3.4665
8 r_o	.75893	.88892	.94384	1.0631
9 $r_o + r_o$	.76692	.89776	.95420	1.0748
10 T_o	.1853	.2115	.2225	.2465
13 $R + \frac{a'}{a}$	.161	.1251	.295	.399
14′ u'	1.56	1.448	1.467	1.41
14 u	1.555	1.51	1.49	1.457
15 *Sum*	1.716	1.761	1.785	1.856
16 W_o	2.797	5.340	6.7705	10.725
17 B	.00628	.01051	.01258	.0186
18 s	1.18	2.00	—	3.13
	1.13	1.73	2.50	3.095
19 k	.0406	.0402	—	.0454
	.0429	.0465	.0385	.0459
t_1				
.196	6.3	3.6	—	1.13
	9.2	5.07	4.15	1.87
.277	5.2	2.40	—	.63
	7.2	3.25	2.60	1.05
.370	3.9	1.66	—	
	5.85	2.53	1.39	
4.77	2.04			
	4.7			

The galvanometer key and the switch S are manipulated by electromagnets, and these are operated from contacts on a pendulum. The switches must be so constructed as to remain in their new position when moved; *e. g.*, S must stay open (or closed) even after the pendulum contact has been passed.

This method of finding β is independent of the damping or time of discharge, as these factors are constant during the determination of the δ_1's and t_1's at any temperature.

The experiment is performed at various temperatures, and the results then summed up. If s is the slope of the straight line represented by (19), modified for common logarithms, we then have as the working equation

$$k = \frac{.4343}{4.19 \times m} \times \frac{B}{s} \text{cal/gm.} \qquad (20)$$

13. PRACTICAL CONSIDERATIONS.

Under equation (16) it was stated that the first form is the better. The fact is that in the experiment the increase in current caused by closing S must be small in order that the approximations used in equations (7) to (14) may hold. Under these conditions di and dr are very small, and the ratio of di/i to dr/r is difficult to get accurately. Call this ratio u. Comparison of the two forms for B shows that

$$u = \frac{1}{2}\left[\frac{f'(T_o)}{a.f(T_o)} - 1\right]. \tag{21}$$

The writer found that the best procedure was to find u from the watts curve, obtained with the lamp burning in the steady state. The slopes to the watts curve were found at a number of points, and the curve of $f'(T)$ was then drawn (see Fig. 12). Then the value of u was computed with known values of $f(T)$, $f'(T)$, and a. It was pointed out in § 9 that $1/a = T - 325$. These values of u may then be plotted against T, and it will be found that u varies linearly, and has a small slope so that we can be pretty sure of our line.

The term $l/(R + l)$ is equal to the ratio of the voltage across the entire lamp to the voltage of the battery. As with a battery whose E.M.F. is much above the voltage of the lamp this ratio is necessarily small, it may be found from voltmeter readings. The ratio $a'/a = (T_o - 525)/(T_o - 325)$, and this can be found with the slide-rule as soon as we know T_o.

The battery voltage will perhaps drop during a set of measurements, but this will not noticeably affect any part of B except the factor W_o. So after a change to a new temperature has been made, it is best to wait a little while until the wiring has assumed the new temperature. Then the throws are obtained as explained. For this part of the experiment it is not necessary to have the potentiometer current correctly adjusted, provided the potentiometer is set so as to balance the lamp voltages. *I. e.*, when the switch S is opened, the potentiometer must be set equal to the final voltage, and similarly for closing S. Then we can measure simply the lower current, the voltage across the apex and across the entire lamp. From these measurements are deduced the watts W_o, the resistances r_o and l_o, and from r_o the temperature T_o is found. The curves in Fig. 12 then give us u and $l/(R + l) \cdot a'/a$, and B can be found. It is not essential even to measure the voltage across the entire lamp if this has been measured during the calibration. The writer plotted a curve for this second term, and thus shortened the time necessary to

complete one set of the throws. In Table VI. it is seen that the current and voltage was also measured with S closed. This was done in order to compare the values of u as found by aid of (21) and as computed directly. It is observed that the latter values are not good.

In this connection it should be stated that as the resistance change R' is so very small, a short-circuiting switch S can not be employed as indicated in Fig. 14. It is necessary, in order to avoid the variation in switch resistance, to use the switch to shunt a resistance of about 8 ohms around a smaller resistance. The amount of this larger resistance depends somewhat on what part of the curve we are working. The change in current should be such that the temperature of the lamp changes by about 20°, and this can be determined from the calibration curves.

14. THE RESULTS.

The results of both methods are plotted in Fig. 13, and distinction is made between results obtained on increasing and decreasing currents. Some of the values by the second method do not occur in such pairs, as in the original try-out the switch S could be only closed by the electromagnet. It is seen that on the whole the second method yields more uniform results. The difference between the heating and cooling methods exists at the lower currents as in Worthing's work, and no explanation is offered other than his own.

Strong doubt is expressed that variation in battery E.M.F. is as serious in the second method as in the first. For if, in the first method, the battery E.M.F. drops, the next point will not belong to the same curve as the preceding points. And if one attempts to correct this by adjusting the rheostats, then again we have different conditions. The first method, especially if not performed by a cyclic arrangement as herein described, takes a very long time, while the second method requires the battery to be steady only about fifteen minutes at a stretch. On the other hand, it is very essential that the rheostats do not change their resistance in the second method, for it is assumed that they remain constant. If the battery voltage is practically equal to the lamp voltage, then variations in rheostat resistance are not so important.

For the second method it was found necessary to use the precision potentiometer, as the type used in the first method can not be set close enough, and also has too high a resistance.

Finally it may be pointed out that β can be found directly by closing the galvanometer key *momentarily* a time t after S has been manipulated,

and adjusting the potentiometer by repeated trials until no throw is observed. But this method is probably no better than the one herein described, and certainly consumes more time.

Further attempts will be made to perfect both methods, and to find the reasons for the difference between the heating and cooling values of k, also to perfect the pyrometric determination of instantaneous temperatures. The very delicate potentiometric measurements being somewhat difficult in the damp summer atmosphere, it was thought best to publish the foregoing results rather than to wait until fall for a continuation of the experiments. Enough results have been obtained to show the possibilities of both methods with improved apparatus.

SUMMARY.

1. A method used and described by Worthing has been slightly altered by using a cyclic method. The chief advantage is a saving of time in making the measurements. Details have been worked out for making the method accurate, and the method has been shown good provided the battery and the driving mechanism function properly.

2. It was shown that unless special precautions are taken, the pyrometric determination of instantaneous temperatures is not practicable. This point is to be taken up later.

3. A variable resistance standard and a deflection potentiometer are described, both of which are desirable in pyrometric work, especially when one has to work alone.

4. A second method has been worked out, differing from Corbino's bridge method in two important details: (1) It is adapted to lamps with potential leads, and (2) the procedure is so modified as to simplify the measurements and calculations. This second method appears to render somewhat more consistent results, although, like the first method, differences in the specific heat appear when computed with increasing and decreasing temperatures, these differences being greater at the lower currents. But the constancy of battery E.M.F. is required over time-intervals of only fifteen minutes, while the first method requires a constant battery for the duration of the entire run.

It is with pleasure that I acknowledge my deep gratitude to the Physics Department of Cornell University for extending to me the courtesies of the Laboratory, and for suggestions and helpful interest. I am no less indebted to the Nela Research Laboratory for assisting me in making

up the lamps; to Dr. Worthing[1] I am especially indebted for his kindness in sharing the problem with me, and for giving me the benefit of his long experience in this kind of work.

CORNELL UNIVERSITY,
ITHACA, N. Y.,
May, 1918.

[1] This paper was practically completed when Worthing's paper on Atomic Heats was sent to the editors of the PHYSICAL REVIEW. Portions of this paper were then changed to prevent duplication.

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